Fiscal Shocks in a Two-Sector Open Economy with Endogenous Markups

Olivier Cardi, Romain Restout

Document de Recherche du Laboratoire d'Économie d'Orléans

DR LEO 2015-21

Fiscal Shocks in a Two-Sector Open Economy with Endogenous Markups

Olivier CARDI
Romain RESTOUT

Laboratoire d'Économie d'Orléans
Collegium DEG
Rue de Blois - BP 26739
45067 Orléans Cedex 2
Tél. : (33) (0)2 38 41 70 37
e-mail : leo@univ-orleans.fr
www.leo-univ-orleans.fr/

FISCAL SHOCKS IN A TWO-SECTOR OPEN ECONOMY WITH ENDOGENOUS MARKUPS*

Olivier CARDI†
Université de Tours LEO (CNRS UMR 7322)
Université de Paris 2 CRED

Romain RESTOUT‡
Université de Lorraine BETA (CNRS UMR 7522)
Université catholique de Louvain IRES

Abstract

We use a two-sector neoclassical open economy model with traded and non-traded goods and endogenous markups to investigate the effects of temporary fiscal shocks. One central finding is that theory can be reconciled with evidence once we allow for endogenous markups and assume that the traded sector is more capital intensive than the non-traded sector. More precisely, while both ingredients are essential to produce the real exchange rate depreciation, only the second ingredient is necessary to account for the simultaneous decline in investment and the current account, in line with the evidence. Keywords: Non-traded Goods; Fiscal Shocks; Investment; Current Account; Endogenous markup.

JEL Classification: F41, E62, E22, F32, L16.

*Constructive comments by Sophie Béreau, Clément Carbonnier, Etienne Lehmann, Jean-Christophe Poutineau, and Stefan Schubert are gratefully acknowledged. An earlier version of this paper was presented to the 12th Spring Meeting of Young Economists held in Hamburg, 25-27th May 2007, the 12th Conference Theories and Methods of Macroeconomics held in Paris, 17-18th January 2008, 14th International Conference on Computing in Economics and Finance held in Paris, 26-28th June 2008, 60th Congress of the French Association of Economics held in Paris, 8-9th September 2011, University of Bolzano lunch seminar, 11th June 2013, Conference on Economic and Financial Adjustments in Europe, Lisbon, 28th June 2013, and has benefited from very helpful comments of participants. Obviously, all remaining shortcomings are ours.

†Corresponding author: Olivier Cardi. Address correspondence: Université Paris 2, CRED, 12 Place du Panthéon, F-75230 Paris Cedex 05. France. Phone: +33 1 44 41 89 64. Fax: +33 1 40 51 81 30. E-mail: olivier.cardi@u-paris2.fr. Univ. Orléans, CNRS, LEO, UMR 7322, F-45067, Orléans, France.

‡Address correspondence : Romain Restout, Université de Lorraine, BETA, 13 place Carnot, CO n°26, F-54035 Nancy Cedex. France. Phone: +33 3 54 50 43 72. Fax: +33 3 54 50 43 51. E-mail : romain.restout@univ-lorraine.fr.

1 Introduction

There has recently been a revival of interest among policy makers in the fiscal policy tool. The fiscal transmission mechanism has also attracted considerable attention in the academic literature. A number of papers have explored the ability of quantitative business cycle models, of both the neoclassical and the new Keynesian variety to account for the data, see e.g. Burnside, Eichenbaum and Fisher [2004], and Gali, Lopez-Salido and Valles [2007], respectively.[1] However, most analyses have been confined to closed economy models and to one-sector frameworks. In the present paper we instead address the following question: to what extent can an open economy version of the two-sector neoclassical model account for the evidence on the fiscal policy transmission mechanism?

Several empirical studies have explored open economy aspects of the fiscal transmission mechanism. One of the most prominent and consistent set of empirical findings that emerges is that a rise in public spending produces a contained increase in GDP, a simultaneous decline in investment and the current account, and most importantly, depreciates the real exchange rate, see e.g. Corsetti et al. [2012], Enders et al. [2011], Monacelli and Perotti [2010].[2] Monacelli and Perotti [2010] show that a New Keynesian model can account for these findings, notably the fall in domestic prices relative to foreign prices, as long as preferences are non separable between consumption and leisure.[3] One major contribution of our paper is to show that the neoclassical model can account for the real exchange rate depreciation along with the simultaneous decline in investment and the current account, once we allow for endogenous markups and the traded sector is assumed to more capital intensive.[4] Our analysis complements Monacelli and Perotti's [2010] study by showing

[1]Hall [2009] compares the predictions of the neoclassical model with those derived from a new Keynesian framework.

[2]Corsetti et al. [2012] use a sample of 17 OECD countries over the period 1975-2008 while Monacelli and Perotti [2010] consider a sample of four countries (Australia, Canada, United Kingdom, United States) over the period running from 1980:1 to 2006:4. Monacelli and Perotti [2010] find a real exchange rate depreciation for Australia, the United States, and the United Kingdom. Enders et al. [2011] corroborate this conclusion for the US while Corsetti et al. [2012] confirm this finding for countries with floating exchange rate regimes.

[3]The intuition behind Monacelli and Perotti's [2010] result is as follows. The authors assume that prices are sticky so that a rise in government spending induces a shift in the labor demand curve because firms have to adjust quantities in face of a demand increase. As a result, the real wage rises. Because the authors allow for non separability in preferences between consumption and leisure, consumption is increasing with the wage rate, although under certain conditions. Under complete markets, consumption risk-sharing implies that the ratio of marginal utilities of consumption are tied to the real exchange rate. By reducing the marginal utility, the rise in consumption produces a fall in domestic prices relative to foreign prices.

[4]We follow Jaimovich and Floetotto [2008] in allowing for the markup to be endogenous. This setup is a multi-sector extension of Linnemann's [2001] model of an endogenous markup. Considering that only

that the combined effect of counter-cyclical markups and inputs reallocation across sectors generates a real exchange rate depreciation after a temporary fiscal expansion in a model with flexible prices, albeit under certain conditions.

Intuitively, whether the traded sector is more or less capital intensive than the non traded sector, resources are shifted toward the non traded sector because public purchases disproportionately benefit this sector.[5] As the rise in government spending boosts non-traded output, profit opportunities trigger the entry of new firms.[6] Hence, the markup falls, regardless of sectoral capital intensities. Because producers with market power mark up prices over the unit cost, the real exchange rate depreciates if both the markup and the unit cost fall or the decline in the markup more than offsets the rise in the unit cost. The change in the unit cost crucially depends on sectoral capital-labor ratio adjustments.[7] When the traded sector is more capital intensive, for a given real exchange rate, the reallocation of inputs keeps sectoral capital-labor ratios fixed so that the unit cost is unaffected. More precisely, a temporary increase in government spending, by implying a rise in future taxes (that we assume to be lump-sum), induces Ricardian agents to increase labor supply which drives down sectoral capital-labor ratios. At the same time, because resources are shifted toward the non traded sector while the traded sector is more capital intensive, capital increases in relative abundance.[8] Hence, the sectoral capital-labor ratios return to their initial values, thus leaving the unit cost for producing unaffected. Because the markup falls, non traded producers set lower prices so that the real exchange rate depreciates, in line with the evidence. Conversely, when the traded sector is more labor intensive, labor rises in relative abundance. Consequently, capital-labor ratios fall dramatically. The return on domestic capital rises which pushes up the unit cost by such an amount that the real exchange rate appreciates (while the markup falls).

By affecting the return on domestic capital, the reallocation of inputs across sectors also

a limited number of intermediate good producers operate in the non-traded sector, the price-elasticity of demand and therefore the markup faced by each firm depends on the number of competitors.

[5] Note that we focus on the effects of a rise in public purchases of non-traded goods because time series of government spending indicate that its non tradable content is substantial, at around 90%.

[6] The substantial increase in non traded output following a rise in government spending is in line with the evidence documented by Benetrix and Lane [2010] who find that the relative size of the non traded sector increases disproportionately after a temporary fiscal shock.

[7] The unit cost function is a weighted average of the wage rate and the rental rate of capital. A fall in sectoral capital-labor ratios has opposite effects on cost components as it raises the rental rate of capital and lowers the wage rate. The latter effect more than offsets the former when the traded sector is more capital intensive so that a fall in sectoral capital-labor ratios lowers the unit cost for producing.

[8] While both inputs are reallocated toward the non traded sector, it is only when the traded sector is more capital intensive that capital increases in relative abundance.

plays a key role in the determination of the responses of investment and the current account. To see this, consider a temporary increase in government spending. As stressed in the classic paper by Baxter and King [1993], households respond to a temporary fiscal expansion by lowering savings, as they try to avoid a large reduction in consumption and/or a large increase in labor supply. Reduced savings imply a decline in investment or the current account, or both. As mentioned above, because the reallocation of inputs across sectors keeps sectoral capital-labor ratios fixed when the traded sector is more capital intensive, the return on domestic capital is unaffected in this case. Hence, the fall in savings drives down both investment and the current account, in line with the evidence. With the reversal of sectoral capital intensities, the two-sector model fails to produce a decline in investment. When the non traded sector is more capital intensive, the large increase in the return of domestic capital induces agents to accumulate physical capital so that investment is crowded in instead of being crowded out.

To address the real exchange depreciation after a temporary fiscal expansion, we draw on earlier work by Turnovsky and Sen [1995] who develop an open economy model with a traded and a non-traded sector, but consider elastic labor supply and imperfect competition in product markets. Coto-Martinez and Dixon [2003] employ a similar framework to investigate the output effects of fiscal shocks, but they restrict their analysis to a permanent rise in public spending and consider fixed markups. When the markups are fixed, the real exchange rate is unaffected if the traded sector is more capital intensive or appreciates with the reversal of sectoral capital intensities, in contradiction with the evidence. Moreover, in contrast to the authors, we analyze the implications of a temporary fiscal expansion. Beyond the fact that considering a transitory increase in public spending allows us to address the VAR evidence, a temporary fiscal shock may lower investment while a permanent fiscal shock always stimulates it.[9]

The remainder of this paper is organized as follows. Section 2 outlines the specification of a two-sector model with traded and non-traded goods. The non-traded sector is assumed to be imperfectly competitive with an endogenous markup. In section 3, we provide an analytical exploration of the effects of temporary fiscal shocks, shedding light on the fiscal transmission with an endogenous markup. In section 4, we report the results of our numerical simulations and assess the ability of the model to account for the evidence. In section 5, we summarize our main results and present our conclusions.

[9]To produce a fall in investment, the fiscal expansion must be temporary in order to induce agents to reduce their savings.

2 The Framework

We consider a small open economy that is populated by a constant number of identical households and firms that have perfect foresight and live forever.[10] The country is small in terms of both world goods and capital markets, and faces a given world interest rate, $r^\star$. A perfectly competitive sector produces a traded good denoted by the superscript T that can be exported and consumed domestically. An imperfectly competitive sector produces a non-traded good denoted by the superscript N which is devoted to physical capital accumulation and domestic consumption.[11] The traded good is chosen as the numeraire.[12]

2.1 Households

At each instant the representative agent consumes traded goods and non-traded goods denoted by C^T and C^N, respectively, which are aggregated by a constant elasticity of substitution function:

$$C\left(C^T, C^N\right) = \left[\varphi^{\frac{1}{\phi}} \left(C^T\right)^{\frac{\phi-1}{\phi}} + (1-\varphi)^{\frac{1}{\phi}} \left(C^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{1}$$

where φ is the weight attached to the traded good in the overall consumption bundle $(0 < \varphi < 1)$ and ϕ is the intratemporal elasticity of substitution $(\phi > 0)$.

The agent is endowed with a unit of time and supplies a fraction $L(t)$ of this unit as labor, while the remainder, $l \equiv 1 - L$, is consumed as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Households decide on consumption and worked hours by maximizing lifetime utility:

$$U = \int_0^\infty \left\{ \frac{1}{1-\frac{1}{\sigma_C}} C(t)^{1-\frac{1}{\sigma_C}} - \gamma \frac{1}{1+\frac{1}{\sigma_L}} L(t)^{1+\frac{1}{\sigma_L}} \right\} e^{-\beta t} \mathrm{d}t, \tag{2}$$

where β is the consumer's discount rate, $\sigma_C > 0$ is the intertemporal elasticity of substitution for consumption, and $\sigma_L > 0$ is the Frisch elasticity of labor supply.

Factor income is derived by supplying labor L at a wage rate W, and capital K at a rental rate R. In addition, they accumulate internationally traded bonds, $B(t)$, that yield

[10] More details on the model as well as the derivations of the results which are stated below are provided in a longer version of the paper with a Technical Appendix available at `http://www.beta-umr7522.fr/productions/publications/2012/2012-17.pdf`.

[11] As stressed by Turnovsky and Sen [1995], allowing for traded capital investment would not affect the results (qualitatively). Furthermore, like Burstein et al. [2004], we find that the non tradable content of investment accounts for a significant share of total investment expenditure (averaging about 60%).

[12] The price of the traded good is determined on the world market and exogenously given for the small open economy.

net interest rate earnings of $r^\star B(t)$. Denoting lump-sum taxes by Z, the households' flow budget constraint can be written as:

$$\dot{B}(t) = r^\star B(t) + R(t)K(t) + W(t)L(t) - Z - P_C\left(P(t)\right)C(t) - P(t)I(t), \tag{3}$$

where P_C is the consumption price index which is a function of the relative price of non-traded goods P. The last two terms represent households' expenditure which includes purchases of consumption goods and investment expenditure PI. Aggregate investment gives rise to overall capital accumulation according to the dynamic equation

$$\dot{K}(t) = I(t) - \delta_K K(t), \tag{4}$$

where we assume that physical capital depreciates at rate δ_K. In the rest of this paper, the time-argument is suppressed in order to increase clarity.

Denoting the co-state variable associated with eq. (3) by λ the first-order conditions characterizing the representative household's optimal plans are:

$$C = \left(P_C\lambda\right)^{-\sigma_C}, \tag{5a}$$

$$L = \left[\left(\lambda/\gamma\right)W\right]^{\sigma_L}, \tag{5b}$$

$$\dot{\lambda} = \lambda\left(\beta - r^\star\right), \tag{5c}$$

$$R/P - \delta_K + \dot{P}/P = r^\star, \tag{5d}$$

plus the appropriate transversality conditions. In an open economy model with a representative agent having perfect foresight, a constant rate of time preference and perfect access to world capital markets, we impose $\beta = r^\star$ in order to generate an interior solution. This standard assumption made in the literature implies that the marginal utility of wealth, λ, will undergo a discrete jump when individuals receive new information and must remain constant over time thereafter, i.e. $\lambda = \bar{\lambda}$.

The homogeneity of $C(.)$ allows a two-stage consumption decision: in the first stage, consumption is determined, and the intratemporal allocation between traded and non-traded goods is decided at the second stage. Applying Shephard's lemma gives $C^N = P_C' C$ where $P_C' = \partial P_C/\partial P$; denoting by α_C the share of non-traded goods in the consumption expenditure, we have $C^N = \alpha_C P_C C/P$ and $C^T = P_C C - PC^N = \left(1-\alpha_C\right)P_C C$.[13]

[13]Specifically, we have $\alpha_C = \frac{(1-\varphi)P^{1-\phi}}{\varphi+(1-\varphi)P^{1-\phi}}$. Note that α_C depends negatively on the relative price P as long as $\phi > 1$.

2.2 Firms

Both the traded and non-traded sectors use physical capital, K^T and K^N, and labor, L^T and L^N, according to Cobb-Douglas production functions $Y^T = \left(K^T\right)^{\theta^T} \left(L^T\right)^{1-\theta^T}$ and $Y^N = \left(K^N\right)^{\theta^N} \left(L^N\right)^{1-\theta^N}$, where θ^T and θ^N represent the capital income share in output in the traded and non-traded sectors respectively. Both sectors face two cost components: a capital rental cost equal to R, and a labor cost equal to the wage rate W. The traded sector is assumed to be perfectly competitive. As described in more detail below, the non-traded sector contains a large number of industries and each industry is composed of differentiated monopolistically competitive intermediate firms.

The final non-traded output, Y^N, is produced in a competitive retail sector with constant-returns-to-scale production which aggregates a continuum measure one of sectoral non-traded goods.[14] We denote the elasticity of substitution between any two different sectoral goods by $\omega > 0$. In each sector, there are $N > 1$ firms producing differentiated goods that are aggregated into a sectoral non-traded good. The elasticity of substitution between any two varieties within a sector is denoted by $\epsilon > 0$, and we assume that this is higher than the elasticity of substitution across sectors, i.e. $\epsilon > \omega$ (see Jaimovich and Floetotto [2008]). Within each sector, there is monopolistic competition; each firm that produces one variety is a price setter. Output $\mathcal{X}_{i,j}$ of firm i in sector j is produced using capital and labor, i.e. $\mathcal{X}_{i,j} = H\left(\mathcal{K}_{i,j}, \mathcal{L}_{i,j}\right)$. Each firm chooses capital and labor by equalizing markup-adjusted marginal products to the marginal cost of inputs, i. e. $PH_K/\mu = R$, and $PH_L/\mu = W$, where μ is the markup over the marginal costs. At a symmetric equilibrium, non-traded output is equal to $Y^N = N\mathcal{X} = H\left(K^N, L^N\right)$ where $L_N = N\mathcal{L}_N$ and $K_N = N\mathcal{K}_N$.

Following Galí [1995], we depart from the usual practice by assuming that the number of firms is large enough so that we can ignore the strategic effects but not so large that the effect of entry on the firm's demand curve is minuscule. Consequently, the price elasticity of demand faced by a single firm is no longer constant and equal to the elasticity of substitution between any two varieties, but rather a function of the number of firms N. Taking into account the fact that output of one variety does not affect the price of final non-traded output, but influences the sectoral price level, in a symmetric equilibrium, the resulting price elasticity of demand is:

$$e\left(N\right) = \epsilon - \frac{\left(\epsilon - \omega\right)}{N}, \quad N \in \left(1, \infty\right). \tag{6}$$

[14] This setup builds on Jaimovich and Floetotto's [2008] framework. Details of its derivation are therefore relegated to the Technical Appendix of the longer version of the paper.

Assuming that $\epsilon > \omega$, the price elasticity of demand faced by one single firm is an increasing function of the number of firms N within a sector. Henceforth, the markup $\mu = \frac{e}{e-1}$ decreases as the number of competitors increases, i.e. $\mu_N < 0$.

We assume instantaneous entry, which implies that the zero-profit condition holds at each instant of time:

$$\pi^N = P\left[\frac{Y^N}{N}\left(1-\frac{1}{\mu}\right)-\psi\right] = 0, \tag{7}$$

where we denote fixed costs by ψ. The zero-profit condition $\pi^N = 0$ can be solved for the number of firms.[15]

Since inputs can move freely between the two sectors, marginal products in the traded and the non-traded sector equalize:

$$\theta^T\left(k^T\right)^{\theta^T-1} = \frac{P}{\mu}\theta^N\left(k^N\right)^{\theta^N-1} \equiv R, \tag{8a}$$

$$\left(1-\theta^T\right)\left(k^T\right)^{\theta^T} = \frac{P}{\mu}\left(1-\theta^N\right)\left(k^N\right)^{\theta^N} \equiv W, \tag{8b}$$

where we denote by $k^i \equiv K^i/L^i$ the capital-labor ratio for sector $i = T, N$,

Aggregating labor and capital over the two sectors gives us the resource constraints for the two inputs:

$$L^T + L^N = L, \quad K^T + K^N = K. \tag{9}$$

2.3 Government

The final agent in the economy is the government which finances government expenditure by raising lump-sum taxes Z in accordance with the balanced condition:

$$G^T + PG^N = Z. \tag{10}$$

Public spending consists of purchases of traded goods, G^T, and non-traded goods, G^N. Since one prominent feature of the time series of government spending is that its non tradable content is substantial, at around 90%, in the following we therefore concentrate on the effects of a rise in public purchases of non-traded goods.

2.4 Short-Run Static Solutions

System (8a)-(8b) can be solved for sectoral capital-labor ratios: $k^T = k^T(P,\mu)$ and $k^N = k^N(P,\mu)$. Using the fact that $W \equiv \left(1-\theta^T\right)\left(k^T\right)^{\theta^T}$, the wage rate also depends on P and

[15] We assume instantaneous entry since the case of no-entry merely affects the results. In a longer version of the paper, we provide analytical and numerical results when imposing no-entry.

μ, i.e. $W = W(P, \mu)$, with $W_P \gtreqless 0$, $W_\mu \lesseqgtr 0$. An increase in the relative price P raises or lowers W depending on whether the traded sector is more or less capital intensive than the non-traded sector. Since a rise in μ produces opposite effects on variables to those induced by a rise in P, we concentrate on the relative price effects to save space.

Plugging sectoral capital-labor ratios into the resource constraints and production functions leads to short-term static solutions for sectoral output: $Y^T = Y^T(K, L, P, \mu)$ and $Y^N = Y^N(K, L, P, \mu)$. According to the Rybczynski effect, a rise in K raises the output of the sector which is more capital intensive, while a rise in L raises the output of the sector which is more labor intensive. An increase in the relative price of non tradables exerts opposite effects on sectoral outputs by shifting resources away from the traded sector towards the non-traded sector.

By substituting first $W = W(P, \mu)$, system (5a)-(5b) can be solved for consumption and labor supply as follows: $C = C\left(\bar{\lambda}, P\right)$ with $C_{\bar{\lambda}} < 0$, $C_P < 0$, and $L = L\left(\bar{\lambda}, P, \mu\right)$ with $L_{\bar{\lambda}} > 0$, $L_P \gtreqless 0$, $L_\mu \lesseqgtr 0$. A rise in the shadow value of wealth induces agents to cut their real expenditure and to supply more labor. By raising the consumption price index, an appreciation in the relative price of non tradables drives down consumption. Finally, depending on whether $k^T \gtreqless k^N$, a rise in P stimulates or depresses labor supply by raising or lowering W.

The zero profit condition (7) can be solved for the number of intermediate producers by inserting first $Y^N = Y^N\left[K, L\left(\bar{\lambda}, P, \mu\right), P, \mu\right]$ (with $\mu = \mu(N)$). We have:

$$N = N\left(K, P, \bar{\lambda}\right), \quad N_K \gtreqless 0, N_P > 0, N_{\bar{\lambda}} \lesseqgtr 0. \tag{11}$$

Since N co-varies with non-traded output Y^N, a rise in P unambiguously stimulates entry while an increase in K (resp. in $\bar{\lambda}$) raises the number of competitors N if the non-traded sector is more (resp. less) capital intensive than the traded sector.

2.5 Macroeconomic Dynamics

We now describe the dynamics. The adjustment of the open economy towards the steady-state is described by a dynamic system which comprises two equations. First, the dynamic equation for the relative price of non-traded goods (5d) equalizes the return on domestic capital and traded bonds $r^\star$. Second, the accumulation equation for physical capital clears the non-traded goods market along the transitional path:

$$\dot{K} = \frac{Y^N(K, L, P, \mu)}{\mu} - C^N\left(\bar{\lambda}, P\right) - G^N - \delta_K K, \tag{12}$$

where $L = L\left(\bar{\lambda}, P, \mu\right)$ and $\mu = \mu\left[N\left(K, P, \bar{\lambda}\right)\right]$.

Linearizing (12) and (5d) which forms a separate subsystem in K and P and assuming that the Jacobian matrix of the differential equation system has one negative eigenvalue denoted by ν_1 and one positive eigenvalue denoted by ν_2, the general solutions for K and P are:

$$K(t) - \tilde{K} = B_1 e^{\nu_1 t} + B_2 e^{\nu_2 t}, \quad P(t) - \tilde{P} = \omega_2^1 B_1 e^{\nu_1 t} + \omega_2^2 B_2 e^{\nu_2 t}, \tag{13}$$

where B_1 and B_2 are constants to be determined and ω_2^i is the element of the eigenvector associated with the eigenvalue ν_i (with $i = 1, 2$). Two features of the two-sector economy's equilibrium dynamics deserve special attention. First, if the markup is fixed and the traded sector is more capital intensive, we have to set $\omega_2^1 = 0$ to rule out unstable paths.[16] Hence, the temporal path for the real exchange rate is flat when $k^T > k^N$.[17] An endogenous markup restores dynamics for the relative price. Specifically, P and K move in opposite directions along a stable path, i.e. $\omega_2^1 < 0$. Second, when the expansionary policy is transitorily implemented (i.e. the fiscal shock only lasts for $\mathcal{T}$ periods), two periods have to be considered, namely a first period (labelled period 1) over which the temporary policy is in effect, and a second period (labelled period 2) after the policy has been removed. While the small country converges towards its new long run equilibrium over period 2, i. e. B_2 must be set to zero, the economy follows unstable paths over period 1. These are described by (13).

Substituting (12) and (10) into (3), we obtain the dynamic equation for the current

[16] With a fixed markup, if $k^T > k^N$, the temporal path for the relative price must be flat for the no-arbitrage condition (5d) to be fulfilled. To see this, suppose that higher demand for non tradables pushes up the price of non traded goods relative to traded goods, i.e. P rises. The real exchange rate appreciation produces a shift of resources toward the non traded sector. Because the traded sector is more capital intensive, capital increases in relative abundance, thus raising the sectoral capital-labor ratios. Hence, the return on domestic capital R falls, thus requiring that $\dot{P} > 0$. A further increase in P raises $k^j \equiv K^j/L^j$ more which lowers the return on domestic capital and thus requires that $\dot{P} > 0$. Hence, the real exchange rate moves away from its steady-state value. To avoid such unstable paths, we have to set $\omega_2^1 = 0$. This point is emphasized by Turnovsky and Sen [1995].

[17] Intuitively, as will become clear later, agents respond to a temporary fiscal expansion by raising labor supply which lowers sectoral capital-labor ratios k^j. At the same time, because resources are shifted toward the non traded sector while the traded sector is more capital intensive, capital increases in relative abundance. Such a reallocation of inputs restores the capital-labor ratios to their initial values. As a result, both the wage rate and the rental rate of capital are unaffected, and so is the unit cost for producing. Consequently, non traded producers do not change their prices. Hence, when the markup is fixed and the traded sector is more capital intensive, a fiscal shock has no effect on the real exchange rate.

account (denoted by $CA \equiv \dot{B}$):

$$\dot{B} = r^{\star}B + Y^T\left(K, L, P, \mu\right) - C^T\left(\bar{\lambda}, P\right) - G^T, \tag{14}$$

where $Y^T - C^T - G^T$ correspond to net exports. Linearizing (14) around the steady-state and substituting (13), the general solution for the stock of foreign assets is given by:[18]

$$B(t) = \tilde{B} + \left[\left(B_0 - \tilde{B}\right) - \Phi_1 B_1 - \Phi_2 B_2\right] e^{r^{\star}t} + \Phi_1 B_1 e^{\nu_1 t} + \Phi_2 B_2 e^{\nu_2 t}. \tag{15}$$

When the disturbance is temporary, we must take into account that assets (i.e. domestic capital and foreign bonds) have been accumulated (or decumulated) over the period 1. The time path for net foreign assets is described by eq. (15) during this unstable period. As stocks of assets are modified over period 1 (i.e. $(0, \mathcal{T})$), we have to take new initial conditions (i.e. $B_{\mathcal{T}}$ and $K_{\mathcal{T}}$) into account when the fiscal policy is removed.

2.6 Steady-State

We now discuss the salient features of the steady-state. Setting $\dot{P} = 0$ into (5d) and using equality of marginal products of labor implying $\frac{\tilde{P}}{\mu(\tilde{N})} = \frac{\Psi^T}{\Psi^N}\left(\frac{\tilde{W}}{\tilde{R}}\right)^{\theta^T - \theta^N}$, we find that the real exchange is positively tied to the markup:

$$\tilde{P} = \Gamma\left[\mu\left(\tilde{N}\right)\right]^{\frac{1-\theta^T}{1-\theta^N}} \tag{16}$$

where $\Gamma > 0$ is a constant equal to $\frac{\Psi^T}{\left(\Psi^N\right)^{\frac{1-\theta^T}{1-\theta^N}}}\left(r^{\star} + \delta^K\right)^{-\frac{\theta^T - \theta^N}{1-\theta^N}}$ with $\Psi^j = \left(\theta^j\right)^{\theta^j}\left(1 - \theta^j\right)^{1-\theta^j}$ $(j = T, N)$.[19] According to (16), a fall in the markup depreciates the real exchange rate in the long run.

Setting $\dot{K} = 0$ into (12) yields the market-clearing condition for the non-traded good:

$$Y^N\left[\tilde{K}, \tilde{L}, \tilde{P}, \mu\left(\tilde{N}\right)\right]/\mu\left(\tilde{N}\right) = C^N\left(\bar{\lambda}, \tilde{P}\right) + \tilde{I} + G^N, \tag{17}$$

where $\tilde{I} = \delta_K \tilde{K}$. Setting $\dot{B} = 0$ into (14) leads to the market-clearing condition for the traded good:

$$Y^T\left[\tilde{K}, \tilde{L}, \tilde{P}, \mu\left(\tilde{N}\right)\right] = -r^{\star}\tilde{B} + C^T\left(\bar{\lambda}, \tilde{P}\right) + G^T. \tag{18}$$

[18]To avoid unnecessary complications, we give analytical expressions for Φ_i (with $i = 1, 2$) when assuming a fixed markup. If $k^T > k^N$, we have $\Phi_1 = -\frac{\tilde{P}\nu_2}{r^{\star} - \nu_1} < 0$ and $\Phi_2 = -\frac{\tilde{P}\nu_1}{r^{\star} - \nu_2}\left\{1 + \frac{\omega_2^2}{\tilde{P}\nu_1}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_1 + \delta_K\right)\right]\right\}$. If $k^N > k^T$, we have $\Phi_1 = -\frac{\tilde{P}\nu_2}{r^{\star} - \nu_1}\left\{1 + \frac{\omega_2^1}{\tilde{P}\nu_2}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\right]\right\}$ and $\Phi_2 = -\frac{\tilde{P}\nu_1}{r^{\star} - \nu_2}$.

[19]Setting $\dot{P} = 0$ yields $\tilde{R} = \tilde{P}\left(r^{\star} + \delta_K\right)$ which can be solved for $\tilde{k}^T$ by using (8a). Inserting the solution for $\tilde{k}^T$ into eq. (8b) yields $\tilde{W} = \left(\Psi^T\right)^{\frac{1}{1-\theta^T}}\left[\tilde{P}\left(r^{\star} + \delta^K\right)\right]^{-\frac{\theta^T}{1-\theta^T}}$. Computing and inserting the ratio $\tilde{W}/\tilde{R}$ into $\frac{\tilde{P}}{\mu(\tilde{N})} = \frac{\Psi^T}{\Psi^N}\left(\frac{\tilde{W}}{\tilde{R}}\right)^{\theta^T - \theta^N}$, and solving for $\tilde{P}$ yields (16).

The number of firms $\tilde{N}$ is determined by the zero profit condition:

$$Y^N \left[\tilde{K}, \tilde{L}, \tilde{P}, \mu\left(\tilde{N}\right)\right] \left[1 - \left(1/\mu\left(\tilde{N}\right)\right)\right] - \tilde{N}\psi = 0. \tag{19}$$

For the country to remain ultimately solvent, we have to impose one single and overall intertemporal budget constraint:[20]

$$B_0 - \tilde{B} = \Phi_1 \left(K_0 - \tilde{K}\right), \tag{20}$$

where $\Phi_1 < 0$ describes the effect of capital accumulation on the external asset position and K_0 and B_0 are the initial conditions.[21] The five equations (16)-(20) jointly determine $\tilde{P}$, $\tilde{K}$, $\tilde{B}$, $\tilde{N}$, and $\bar{\lambda}$.

3 Temporary Fiscal Expansion: An Analytical Exploration

In this section, we analytically explore the short-run effects of a fiscal expansion. As the shocks identified in the VAR literature are transitory, in this paper we focus the theoretical analysis on temporary increases in government spending. We suppose that at time $t = 0$, the government raises public spending on the non-traded good and at time $\mathcal{T}$ it removes the expansionary budget policy.[22] The higher $\mathcal{T}$, the greater the persistence of the shock.[23]

Our model has two distinctive features: the two-sector dimension and imperfectly competitive product markets with endogenous markups. In assessing the ability of the model with tradables and non tradables to account for the evidence, we adopt a two-step approach. First, we emphasize how an endogenous markup produces the real exchange rate depreciation documented by the empirical literature on the effects of fiscal shocks. To do so, we derive a number of analytical results by abstracting from physical capital. We then discuss the implications of introducing physical capital in the setup. This allows us to explain how allowing for endogenous markups is a necessary but not sufficient condition to produce a

[20] By first substituting the short-run solutions, then linearizing the dynamic equation of the internationally traded bonds (14) in the neighborhood of the steady-state, substituting the solutions for $K(t)$ and $P(t)$ and finally invoking the transversality condition, we obtain the linearized version of the nation's intertemporal budget constraint (20).

[21] Since for all parameterizations, Φ_1 is always negative, we assume $\Phi_1 < 0$ from now on. Hence, capital accumulation deteriorates the current account along the transitional path.

[22] We assume further that at the outset all agents perfectly understand the temporary nature of the policy change. Hence, at time $\mathcal{T}$, there is no new information and thus no jump in the marginal utility of wealth at this date.

[23] To derive formal solutions after a temporary fiscal shock, we applied the procedure developed by Schubert and Turnovsky [2002].

real exchange rate depreciation. Second, because the simultaneous decline in investment and the current account is one of the most consistent responses to a fiscal shock documented in the empirical literature, we analytically assess the ability of the two-sector sector model to account for this finding. Because considering counter-cyclical markups is not essential to produce a crowding out of investment and a current account deficit following a temporary fiscal expansion, we provide analytical results with a fixed markup by assuming that the number within each non traded industry is large so that the number of competitors has no effect on the markup.[24]

We first explore the implications of an endogenous markup in a two-sector open economy without physical capital; the main result is that the real exchange rate unambiguously depreciates after a temporary fiscal shock in a model abstracting from physical capital accumulation. We assume constant returns-to-scale technology, i.e. $Y^T = L^T$ and $Y^N = L^N$. Due to perfect labor mobility across sectors, we have $1 = P/\mu = W$ where the markup $\mu = \mu\left(N\right)$ decreases as the number of competitors N increases. Imposing free-entry, the zero-profit condition can be solved for the number of firms, i.e. $N = N\left(L^N\right)$. The resource constraint for labor reads $L = L^T + L^N$. First-order conditions (5a)-(5c) hold. The non-traded good market clearing condition can be rewritten as follows $L^N/\mu = C^N + G^N$ while the traded good market clearing condition is $r^\star \tilde{B} + L^T = C^T + G^T$. The intertemporal solvency condition now reduces to $\tilde{B} = B_0$.[25]

We now investigate the effects of a temporary rise in G^N. To begin with, in a model abstracting from physical capital accumulation, all variables adjust instantaneously to their long-run levels, except the stock of foreign assets; hence, the tilde is suppressed in order to increase clarity. By raising (lump-sum) taxes to balance the budget and reducing households' disposable income, a fiscal expansion produces an increase in the shadow value of

[24]Unfortunately, a two-sector model with endogenous markups and physical capital accumulation is analytically untractable.

[25]While $\tilde{B}$ remains unaffected after permanent fiscal shocks, a temporary rise in G^N unambiguously lowers $\tilde{B}$ in the long-run.

wealth as shown formally below:[26]

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \frac{P\bar{\lambda}}{Y\chi}\frac{\left[1+\frac{\omega_C(1-\alpha_C)\alpha_C}{\omega_N}\left(\phi-\sigma_C\right)\eta_{\mu,N}\eta_{N,L^N}\right]}{\Psi}\left(1-e^{-r^\star T}\right) > 0, \tag{21}$$

where $\chi < 0$, $\Psi > 0$, $\eta_{\mu,N} < 0$ is the elasticity of the markup to the number of competitors, and $\eta_{N,L^N} > 0$ the elasticity of the number of competitors to non-traded labor.[27]

The negative wealth effect induces agents to work more and cut real expenditure. Because the decline in real expenditure is spread over the two goods, the rise in G^N more than offsets the fall in C^N so that labor in the non-traded sector unambiguously rises:[28]

$$\left.\frac{\mathrm{d}L^N(0)}{\mathrm{d}G^N}\right|_{temp} = -\frac{\omega_C\alpha_C\sigma_C L^N}{\bar{\lambda}\chi\omega_N}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + \frac{P}{\chi} > 0. \tag{22}$$

Higher non-traded output creates profit opportunities, inducing new firms to enter the market. Hence, the markup μ unambiguously falls. Because $P = \mu\, W$ and perfect mobility of labor implies that $W = 1$, non-traded producers with market power set lower prices as they perceive a more elastic demand. As a result, the real exchange rate depreciates, in line with the evidence.

Introducing physical capital implies that the real exchange rate does not necessarily depreciate. Below, we emphasize that we have to assume that the traded sector is more capital intensive to generate a real exchange rate depreciation. Specifically, non traded producers with market power mark up prices over the unit cost UC, i.e. $P = \mu\ UC$. The unit cost for producing one unit of the non-traded good is a weighted average of the rental rate of capital R and the wage rate W, i.e. $UC = \frac{R^{\theta^N}W^{1-\theta^N}}{(\theta^N)^{\theta^N}(1-\theta^N)^{1-\theta^N}}$. For the real exchange rate to depreciate, both the markup and the unit cost must fall or alternatively the decline in the markup must offset the rise in the unit cost. The adjustment in the unit cost crucially depends on sectoral capital-labor ratios changes. To see it, take logarithm and differentiate $P = \mu\ UC$, and use the fact that $\hat{R} = -\left(1-\theta^T\right)\hat{k}^T$ and $\hat{W} = \theta^T\hat{k}^T$;

[26]The term $\Psi \equiv \sigma_L + \sigma_C\left(1-\alpha_C\right)\omega_C + \frac{\alpha_C\omega_C\sigma_C}{\chi}\left[1+\frac{(1-\alpha_C)\omega_C}{\omega_N}\alpha_C\left(\phi-\sigma_C\right)\eta_{\mu,N}\eta_{N,L^N}\right] > 0$, with $\chi = 1-\eta_{\mu,N}\eta_{N,L^N}\left\{1-\frac{\alpha_C\omega_C}{\omega_N}\left[\left(1-\alpha_C\right)\phi+\alpha_C\sigma_C\right]\right\} > 0$, is a function of $\eta_{\mu,N} < 0$, $\eta_{N,L^N} > 0$, the ratio of consumption expenditure to GDP ($\omega_C = \frac{P_C C}{Y}$), and the ratio of non-traded output less fixed costs to GDP ($\omega_N = \frac{PY^N/\mu}{Y} = \frac{L^N}{L}$). Note that Ψ and χ are positive as long as the markup is not too responsive to the entry of firms, i.e., $\eta_{\mu,N} < 0$ is not too large. This condition always holds for reasonable values of the markup μ.

[27]Since the increase in G^N is only temporary, the present value of the necessary tax increases to satisfy the government's intertemporal budget constraint is less than that of an equal but permanent increase in G^N, as captured by the scaling-down term $0 < \left(1-e^{-r^\star T}\right) < 1$ in eq. (21).

[28]The positive influence of higher G^N on L^N is captured by the second term on the RHS of (22). Because the wealth effect is smaller after a temporary rise in G^N than after a permanent fiscal shock, the first term on the RHS of (22) is less negative and hence L^N increases more in the former case (since C^N falls less).

denoting the percentage deviation from its initial steady-state by a hat and rearranging terms, we get:

$$\hat{P} = \hat{\mu} + \left(\theta^T - \theta^N\right) \hat{k}^T, \tag{23}$$

where $\hat{k}^T = \hat{k}^N$. According to (23), the markup decline yields a real exchange rate depreciation if the capital-labor ratios k^j (with $j = T, N$) remain fixed because in this case, the unit cost is unaffected. While a higher labor supply lowers $k^j = K^j/L^j$ on impact, the reallocation of inputs across sectors may keep the sectoral capital-labor ratios unchanged. More precisely, while both inputs are shifted toward the non-traded sector, capital increases in relative abundance when the traded sector is more capital intensive. In this case, for given P, the reallocation of inputs keeps capital-labor ratios k^j fixed, thus leaving the unit cost unchanged. Eq. (23) implies that the fall in the markup lowers the real exchange rate. Conversely, when the non-traded sector is more capital intensive, labor increases in relative abundance, thus lowering further the capital-labor ratios. As shown by eq. (23), if $\theta^N > \theta^T$, reduced capital-labor ratios k^j raise the unit cost by raising the rental rate of capital. As a result, the real exchange rate may appreciate instead of depreciating. As discussed in the next section, we find numerically that the rise in the unit cost pushes up the real exchange rate (while the markup falls) across all scenarios when $k^N > k^T$.

We now turn to the responses of investment and the current account following a temporary fiscal expansion. In order to preserve analytical tractability, we assume that the number of competitors within each industry is large enough so that eq. (6) implies that the price-elasticity of demand e reduces to ϵ; in this case, the markup is fixed, i.e. $\mu = \frac{\epsilon}{\epsilon-1}$. To begin with, it should be mentioned that an endogenous markup is not a key ingredient to produce either a crowding out of investment or a current account deficit.[29] The reallocation of inputs plays a key role instead in the determination of investment and the current account responses. To avoid unnecessary complications, we provide analytical results for the initial responses for investment and the current account when the traded sector is more capital intensive than the non-traded sector.[30] Formal expressions allow us to analyze the role of the length of the fiscal shock captured by $\mathcal{T}$ and of the elasticity of labor supply σ_L.

The response of investment is the result of two opposite effects. On the one hand, according to Rybczynski's theorem, a rise in labor supply raises the output of the sector which is more labor intensive and thus stimulates investment. On the other hand, because

[29] When discussing numerical results in the next section, we emphasize how an endogenous markup modifies the responses of investment and the current account. As will become clear later, the fall in the markup plays a secondary role in the adjustment of investment and the current account.

[30] Analytical results when $k^N > k^T$ can be found in the longer version of the paper.

the decline in real expenditure is spread over the two goods, the fall in C^N is not large enough to more than offset the rise in G^N which exerts a negative effect on investment. Hence, higher public spending G^N may crowd in or crowd out capital investment. Formally, the initial reaction of investment is ambiguous:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} = -\left\{1 + \left(1 - e^{-r^\star \mathcal{T}}\right) \frac{\left[\sigma_L \tilde{L}\tilde{k}^T \tilde{P}\left(\nu_1 + \delta_K\right) - \sigma_C \tilde{P}\tilde{C}^N\right]}{\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)}\right\} \lesseqgtr 0. \tag{24}$$

When setting $\sigma_L = 0$ in (24), the reaction of investment becomes unambiguously negative, i.e. $\mathrm{d}I(0)/\mathrm{d}G^N = \alpha_C\left(1 - e^{-r^\star \mathcal{T}}\right) - 1 < 0$, because the rise in public spending G^N exceeds the fall in C^N. As long as $\sigma_L > 0$, the sign of (24) is no longer clear-cut. The less responsive the labor supply (i.e. the smaller σ_L) or the shorter the fiscal expansion (i.e. the lower $\mathcal{T}$), the more likely it is that investment is crowded out by public spending.[31] When the fiscal shock is short-lived, the negative wealth effect is smaller so that labor supply and hence Y^N increases less while consumption of non-tradables C^N falls by a smaller amount.

Turning to the initial response of the current account, we obtain after computation:

$$\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} = -\tilde{P}e^{-r^\star \mathcal{T}} + \tilde{P}\left[1 + \left(1 - e^{-r^\star \mathcal{T}}\right) \frac{\left[\sigma_L \tilde{L}\tilde{k}^T \tilde{P}\left(\nu_1 + \delta_K\right) - \sigma_C \tilde{P}\tilde{C}^N\right]}{\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)}\right] \lesseqgtr 0. \tag{25}$$

The first term on the RHS of (25) represents the negative impact of reduced savings on the current account. The second term on the RHS of (25) represents the influence of investment on the net foreign asset position. When setting σ_L to zero into (25), the initial current account response, given by $\mathrm{d}CA(0)/\mathrm{d}G^N = \tilde{P}\left(1 - \alpha_C\right)\left(1 - e^{-r^\star \mathcal{T}}\right)$, becomes unambiguously positive. The reason is that the decline in investment is large enough to more than offset the drop in private savings induced by the smoothing behavior. The more responsive the labor supply (i.e. σ_L is higher), the smaller the decline in investment on impact, and hence the more likely it is that the open economy experiences a current account deficit. The length of the shock $\mathcal{T}$ exerts two opposite effects on the initial response of the current account. On the one hand, as the fiscal shock is shorter (i.e. $\mathcal{T}$ becomes smaller) agents are more willing to reduce private savings which amplifies the deterioration in the net foreign asset position. On the other hand, investment declines more which exerts a positive effect on the current account. The overall effect will be determined numerically.

When the non-traded sector is more capital intensive, investment is less likely to be crowded out by public spending than if $k^T > k^N$. In the latter case, the reallocation of

[31] More precisely, when $\mathcal{T}$ is smaller than the critical date $\hat{\mathcal{T}} = -\frac{1}{r^\star} \ln\left[\frac{\left(\sigma_C \tilde{C}^T - \sigma_L \tilde{L}\tilde{k}^N \tilde{P}\nu_2\right)}{\left(\sigma_L \tilde{L}\tilde{k}^T \tilde{P}(\nu_1 + \delta_K) - \sigma_C \tilde{P}\tilde{C}^N\right)}\right]$, then investment is crowded out by public spending.

inputs keep sectoral capital-intensities fixed so that the real exchange rate remains unaffected by the fiscal shock. Conversely, when the non-traded sector is more capital intensive, the real exchange rate appreciates because the unit cost for producing rises. By shifting resources toward the non-traded sector, the rise in P has an expansionary effect on non-traded output and thus on investment.

4 Temporary Fiscal Expansion: A Quantitative Exploration

In this section, we quantitatively analyze the effects of a temporary rise in government spending. For this purpose we numerically solve the open economy model with endogenous markups. In the following we thus first discuss parameter values before turning to the short-term effects of the fiscal shock.

4.1 Baseline Parametrization

Since we calibrate a two-sector model with tradables and non-tradables, we pay particular attention to the suitability of the non-tradable content of the model to the data. Table 1 summarizes the non-tradable content of GDP, employment, consumption, and government spending, and gives the share of government spending on the traded and non-traded good in the sectoral output, the shares of capital income in output in both sectors, and the markup charged by the non-traded sector for all countries of our sample.[32]

We start by describing the calibration of consumption-side parameters which we use as a baseline. The world interest rate which is equal to the subjective time discount rate β is set to 1%. One period of time corresponds to a quarter. The elasticity of substitution between traded and non-traded goods ϕ is set to 1.5 (see e.g. Cashin and Mc Dermott [2003]). The weight φ of consumption of tradables is set to 0.5 in the baseline calibration to target a non-tradable content in total consumption expenditure (i.e. α_C) of 45%, in line with our estimates. The intertemporal elasticity of substitution for consumption σ_C is set to 0.5 because empirical evidence overwhelmingly suggests values smaller than one. One critical parameter is the intertemporal elasticity of substitution for labor supply σ_L. In our baseline parametrization, we set $\sigma_L = 0.5$, in line with evidence reported by Domeij and Flodén [2006].

We now describe the calibration of production-side parameters. We assume that physical

[32] Our sample covers thirteen OECD economies over the period 1970-2004. Targeted ratios when calibrating are the thirteen OECD countries' unweighted average.

capital depreciates at a rate $\delta_K = 1.5\%$ to target an investment-GDP ratio of 20%. The shares of sectoral capital income in output take two different values depending on whether the traded sector is more or less capital intensive than the non-traded sector. In line with our estimates, if $k^T > k^N$, θ^T and θ^N are set to 0.4 and 0.3, respectively.[33] Alternatively, when $k^N > k^T$, we choose $\theta^T = 0.3$ and $\theta^N = 0.4$. Setting the elasticity of substitution between sectoral goods, ω, to 1 and the elasticity of substitution between varieties, ϵ, to 4 yields a markup μ charged by the non-traded sector of 1.35, which is close to the thirteen OECD countries' unweighted average.

We set G^N and G^T so as to yield a non-tradable share of government spending of 90%, and government spending as a share of GDP of 20%.[34] We consider three different scenarios for the duration of the fiscal shock: a short-lived ($\mathcal{T} = 8$), a medium-lived ($\mathcal{T} = 16$), and a long-lived ($\mathcal{T} = 32$) fiscal shock. As the baseline scenario, we take the medium-lived fiscal shock, i.e. a shock that lasts 16 quarters. In this case, the cumulative increase in government spending corresponds approximately to the cumulative increase in US government spending six years after an exogenous spending shock by one percentage point of GDP according to the estimates reported by Cardi and Müller [2011]. For $\mathcal{T} = 16$, we also conduct a sensitivity analysis with respect to the elasticity of labor supply (i.e. we set σ_L to 0.1 and 1).

< Please insert Table 1 about here >

4.2 Short-Run Effects

We now discuss the short-run effects of the fiscal expansion. We take the medium-lived spending shock as our baseline scenario, but we also refer to short-lived and long-lived fiscal shocks, as the length of fiscal stimulus may vary across countries. Panels A and B of Table 2 show the results for this situation, as well as for a number of alternative scenarios. While panel A gives the response on impact, panel B displays the cumulative responses over the first two years (i.e. eight quarters) after the shock. In order to emphasize how an

[33] Table 1 gives the values of θ^j $(j = T, N)$. The values of θ^T and θ^N we have chosen correspond roughly to the averages for countries with $k^T > k^N$. For these values, the non-tradable content of GDP and labor are 63% and 66%, respectively. When $k^N > k^T$, we can use reverse but symmetric values for θ^N so that the size of $k^T - k^N$ remains unchanged. For $\theta^T = 0.3$ and $\theta^N = 0.4$, the non-tradable contents of GDP and labor are 69% and 65%, respectively.

[34] Close to the average of the values reported in Table 1, the ratios G^T/Y^T and G^N/Y^N are 6% and 28% in the baseline calibration.

endogenous markup improves the predictive power of the two-sector model, we numerically solve the model with a fixed markup as well. Numerical results for a fixed markup are shown in the first column when $k^T > k^N$ and in the seventh column with the reversal of sectoral capital intensities.

Before analyzing in detail the role of counter-cyclical markups and sectoral reallocation in shaping the short-run dynamics in response to a temporary increase in government spending, we should mention the set of empirical evidence established by Corsetti et al. [2012] which confirms earlier findings by Monacelli and Perotti [2010]. Using a sample of seventeen OECD countries over the period 1975-2008, it is found that an exogenous increase in government spending moderately raises output, induces a simultaneous decline in investment and the current account and depreciates the real exchange rate. In the following, we discuss the predictions of our model for the behavior of these variables when $k^T > k^N$ and when $k^N > k^T$.

We first address the response of the real exchange rate to a temporary fiscal expansion. As shown in panel A of Table 2, P drops on impact across all scenarios, as long as the markup is endogenous and the traded sector is more capital intensive. When $k^T > k^N$, the cumulative responses shown in panel B reveal that the real exchange rate depreciates by 0.11% for the baseline scenario while P remains unaffected if the markup is fixed (see the first column). The intuition behind this result is as follows. By producing a negative wealth effect, a fiscal expansion induces agents to supply more labor, which lowers sectoral capital-labor ratios. At the same time, because resources are shifted toward the non traded sector while the traded sector is more capital intensive, capital increases in relative abundance. As a result, the reallocation of inputs restores the sectoral capital-labor ratios to their initial values. Hence, the unit cost of producing is unaffected so that non traded producers do not change their prices along the transitional path. While this chain of events holds when markups are counter-cyclical, the shift of resources toward the non-traded sector and the resulting increase in non-traded output produces an additional channel through which government spending influences the real exchange rate. More precisely, because a temporary fiscal shock has an expansionary effect on non-traded output, profit opportunities trigger firm entry which reduces the markup and thus induces non traded producers to set lower prices. Consequently, if the traded sector is more capital intensive and the markup is endogenous, the real exchange rate depreciates in all scenarios following a fiscal expansion, as shown in panel B of Table 2.

Conversely, whether the markup is fixed or endogenous, the real exchange rate appreciates when $k^N > k^T$, as shown in panel A and panel B of Table 2. In this case, resources move toward the non-traded sector while the traded sector is more labor intensive. Because labor increases in relative abundance, sectoral capital-labor ratios fall. The rental rate of capital and thus the unit cost of producing rises, which induces non-traded producers to set higher prices. Hence, the real exchange rate appreciates in this configuration, although the markup falls when μ is endogenous.

We now turn to the output effects of a fiscal expansion. While employment and thus GDP increases in all the scenarios where $k^T > k^N$, labor supply and output rise slightly or decrease when the sectoral capital intensities are reversed. The reason is that when $k^T > k^N$, agents are induced to supply more labor as a result of the wealth effect. By contrast, when $k^N > k^T$, the appreciation of the real exchange rate drives down the wage rate by reducing sectoral capital-labor ratios, which in turn counteracts the wealth effect. When contrasting the output effect in the case of an endogenous markup with that in the case of a fixed markup, panel A of Table 2 shows that a counter-cyclical markup moderates the increase in GDP. The reason is that sectoral capital-labor ratios fall slightly when $k^T > k^N$ and drop dramatically if $k^N > k^T$.[35] Hence, with an endogenous markup, the wage rate declines, regardless of sectoral capital intensities, which in turn moderates the rise in labor supply, and thus the expansionary effect on GDP.

In line with our theoretical predictions, we find numerically that the short-run response of investment depends heavily on sectoral capital intensities. On impact, an increase in G^N crowds out investment only if the traded sector is more capital intensive. In this case, while non-traded output expands as a result of the increase in labor supply, the rise in public spending G^N produces an excess of demand which must be eliminated by a drop in investment. As shown in the seventh line of panel A of Table 2, the less elastic labor supply is, the larger the crowding-out of investment. Moreover, as discussed in section 3, when the length of the fiscal shock increases (i.e. $\mathcal{T}$ is higher), consumption falls more because the negative wealth effect is larger so that investment declines less. For $k^T > k^N$, the cumulative responses reported in the third line of panel B of Table 2 show that a fiscal expansion crowds out investment by 3.16% of initial GDP if the markup is fixed and by 3.55% if the markup is endogenous. The larger drop in investment when markups are counter-cyclical stems from the real exchange rate depreciation which exerts a negative

[35]It should be mentioned that when the traded sector is more capital intensive, sectoral capital-labor ratios only fall if the markup is endogenous while $k^j \equiv K^j/L^j$ remain unaffected if the markup is fixed. More precisely, for given P, the reallocation of inputs keeps k^j unchanged. Because P falls with an endogenous markup, resources move toward the traded sector; capital-labor ratios k^j drop by a small amount though.

impact on non-traded output and thus on physical capital accumulation.

By contrast, if the non-traded sector is more capital intensive, the increase in G^N triggers an appreciation in the relative price of non tradables P which stimulates Y^N and hence investment, in all scenarios. The cumulative responses reported in the third line of panel B of Table 2 show that a fiscal expansion crowds in investment by 3.22% if the markup is fixed and 4.28% of initial GDP when the markup is endogenous. A counter-cyclical markup amplifies the rise in investment following a fiscal shock when $k^N > k^T$ because the entry of new firms induces non-traded producers to raise output further (i.e. Y^N rises more) as they perceive a more elastic demand.

As shown in the eighth line of panel B of Table 2, the open economy experiences a current account deficit, regardless of sectoral capital intensities.[36] In both cases, agents smooth consumption by reducing private savings, which in turn deteriorates the net foreign asset position. When $k^T > k^N$, the decline in the current account triggered by the fall in savings is moderated by the drop in investment. The current account deficit after two years shrinks from 3.91% with a fixed markup of initial GDP to 3.61% with an endogenous markup because in the latter case, investment falls more. As shown in panel B of Table 2, in line with our theoretical predictions, the more responsive the labor supply is, the larger the current account deficit. Moreover, while analytically, increasing the length of the fiscal shock captured by $\mathcal{T}$ has an ambiguous effect on the size of the current account deficit because it moderates the decline in both savings and investment, numerical results reveal that the latter effect predominates so that the net foreign asset position deteriorates more when raising $\mathcal{T}$ from $\mathcal{T} = 16$ to $\mathcal{T} = 32$.[37] When the non-traded sector is more capital intensive, as shown in panel B of Table 2, the open economy runs a substantial current account deficit as savings fall while investment rises.

< Please insert Table 2 about here >

---

[36] It is worth noting that the current account begins to show a surplus on impact when the markup is endogenous while the current account falls immediately if the markup is fixed, as shown in panel A of Table 2. The reason is that in the former case, investment falls dramatically. However, the current account deteriorates rapidly.

[37] Cardi and Müller [2011] provide point estimates for GDP, investment and current account responses after an exogenous increase in government spending by one percentage point of GDP. When comparing our results when $k^T > k^N$ (panel B of Table 2) with the numbers reported by Cardi and Müller [2011], we find that our two-sector model tends to overpredict both the crowding out of investment and the current account deficit, while it predicts the GDP response pretty well.

< Please insert Figure 1 about here >

We have also computed the impact and cumulative responses of the real consumption wage, i.e. W/P_C. The analysis of the adjustment of the real wage is of particular interest since a number of empirical studies, specifically Pappa [2009] and Perotti [2007] for the US and Benetrix [2012] for eleven Euro area countries, find that the real consumption wage increases following a rise in government spending while the standard one-sector open economy model predicts the opposite. The reason is that agents supply more labor, which lowers the capital-labor ratio and therefore the real wage. By contrast, a two-sector neoclassical model may produce an increase in the real consumption wage due to the combined effect of the reallocation of inputs and of a counter-cyclical markup. To see it, let us assume that the traded sector is more capital intensive. In this configuration, the reallocation of inputs prevents the wage rate from decreasing by keeping the sectoral capital-labor ratios fixed for given P. Moreover, by producing a real exchange rate depreciation and thus a fall in the consumption price index, a decline in the markup may push up the real consumption wage. By contrast, when the non-traded sector is more capital intensive, a fiscal shock unambiguously lowers the real consumption wage by reducing the wage rate W and appreciating the real exchange rate.

In the light of our discussion above, we concentrate on the most interesting case, i.e. $k^T > k^N$. As shown in the fourth line of panel A of Table 2, if the traded sector is more capital intensive, the decline in the wage rate more than offsets the drop in the consumption price index so that the real consumption wage decreases on impact in all scenarios. While the reallocation of inputs keep capital-labor ratios k^j fixed for given P, the real exchange rate depreciation induces a shift a resources toward the traded sector, thus reducing k^j. While the capital-labor ratios fall by a small amount, the consecutive drop in W is large enough to drive down the real wage. The second line of panel B shows that the two-year horizon cumulative response of the real wage is negative for the baseline scenario. Only if the fiscal shock is short-lived or long-lived (i.e., G^N is raised over 8 or 32 quarters) does the cumulative response of the real wage become positive, as displayed in the second line of panel B of Table 2. After a long-lived fiscal shock, both non-traded output expansion and, as a consequence, firm entry are larger. Hence, the decline in the markup is large enough to produce a positive cumulative response of the real wage. Following a short-lived fiscal shock, the real exchange rate appreciates rapidly after its short-term depreciation, and it has a positive impact on the wage rate by raising the capital-labor ratios.

The last dimension of the fiscal policy transmission that our model highlights is the sectoral effects of a rise in government spending. Only a few previous studies have estimated the effects of a boost to government spending on sectoral outputs. Among these, Bénétrix and Lane [2010] find that fiscal spending shocks generate a shift in the sectoral composition of output as public purchases disproportionately benefit the non-traded sector. As summarized in the last two lines of panels A and B of Table 2, across all the scenarios, we find numerically that a rise in government spending boosts non-traded output, more so if the non-traded sector is more capital intensive.

4.3 Transitional Adjustment

We now discuss the dynamic effects. The transitional paths of key variables under the baseline and alternative scenarios are displayed in Figure 1. The responses of GDP, investment and current account are expressed as a percentage of the initial steady-state output, while the real exchange rate and the real wage are given as the percentage deviation from the initial steady state. Horizontal axes measure quarters. The solid line gives results for an endogenous markup and the dashed line for a fixed markup.

The transitional path of investment is quite distinct, depending on whether the traded sector is more or less capital intensive than the non-traded sector. Along the transitional path, capital accumulation clears the non-traded good market. When $k^T > k^N$, the size of the crowding-out of investment falls over time, but when $k^N > k^T$, investment decreases monotonically as the depreciation in the relative price P (after its initial appreciation) lowers non-traded output. After about 2 years, the investment flow becomes negative and the open economy decumulates physical capital until the fiscal policy is removed. At time $\mathcal{T}$, government spending G^N reverts back to its initial level which releases resources for capital accumulation. Regardless of sectoral capital intensities, investment is crowded in.

The temporal path for GDP is driven by the adjustments in both labor and capital. In the case $k^T > k^N$, because labor increases less when the markup is endogenous than if the markup is fixed, GDP rises by a smaller amount on impact. The dynamics for GDP are the mirror image of capital accumulation: the slowdown in GDP growth as government spending is raised originates from the crowding out of investment. By contrast, when $k^N > k^T$, the temporal path of output is hump-shaped: GDP growth first increases as labor supply rises, and then slows down as a result of the negative investment flow which starts after about two years. At the time the fiscal policy is removed, the economy experiences an investment boom which boosts GDP in both cases when the markup is fixed.

Regardless of sectoral capital intensities, the current account stays in deficit while government spending is raised. In the case $k^T > k^N$, the decumulation of foreign bonds reflects the negative impact of consumption smoothing behavior on the current account, even though the crowding out of investment counteracts this effect. If the sectoral capital intensities are reversed, the depreciation in the relative price of non tradables reduces investment, which exerts a positive impact on the current account. Yet in the latter case, the current account deficit at a horizon of two years is almost three times larger than if $k^T > k^N$, as shown in the fourth line of panel B of Table 2.

As shown in the fifth row of Figure 1, if the traded sector is more capital intensive, the real exchange rate falls when the markup is endogenous while the dynamics for P degenerate when the markup is fixed. In the former case, the dynamics of the real exchange rate are U-shaped.[38] When $k^T > k^N$, the fifth row of Figure 1 indicates that the real exchange appreciates after eight quarters but P remains below its initial level. As displayed in the last row of Figure 1, while the real exchange rate depreciation is not large enough to push the real wage on impact, the dynamic path for W/P_C reveals that it increases along the transitional path and exceeds its initial level after about 6 quarters.[39]

5 Conclusion

A robust conclusion emerging from empirical papers is that government spending tends to depreciate the real exchange rate and to crowd out both investment and the current account. In this paper, we build a neoclassical model with counter-cyclical markups that can simultaneously match all these facts. We show that the open economy version of the two-sector neoclassical model with traded and non-traded goods can account for the empirical evidence on the effects of fiscal shocks, but only if markups are endogenous and the traded sector is more capital intensive than the non-traded sector. While both ingredients

[38] The reason is as follows. The real exchange rate depreciation shifts resources towards the traded sector. Since the non-traded sector is more labor intensive, the sectoral capital-labor ratios fall, which in turn raises the return on domestic capital. According to (5d), for the no-arbitrage condition to hold, the real exchange rate must decline over time, i.e. $\dot{P}/P < 0$. However, the fall in μ exerts opposite effects on R. At a certain date, the decline in the markup is large enough to offset the impact of the relative price on the return of capital.

[39] While the initial fall in the real consumption wage does not conform to the findings by Perotti [2007] and Benetrix [2012], it should be mentioned that the dynamics for W/P_C accommodate pretty well the evidence documented by Ramey [2011] who finds that the real wage falls on impact, rises along the transitional path and exceeds its initial level after four or eight quarters, depending on the period considered.

are essential to account for the real exchange rate depreciation, only the second ingredient is necessary to account for the simultaneous decline in investment and the current account.

We also find that our model may produce the positive response of the real wage documented by Perotti [2007] for the U.S. and more recently by Benetrix [2012] who use a sample of eleven Euro area countries. In our model, the rise in the real consumption wage stems from the real exchange rate depreciation which lowers the consumption price index while the wage rate falls by a small amount. Our numerical results reveal that the real consumption wage increases only if the fiscal shock is short- or long-lived, not if it holds for a medium term.

In conclusion, we must stress a number of caveats. If the non-traded sector is assumed to be the more capital intensive sector, the model fails to match the evidence along a number of dimensions. Notably, in this case, the two-sector model cannot account for the crowding-out of investment which is one of the most consistent responses to a fiscal shock documented in the empirical literature. Additionally, if the traded sector is more capital intensive than the non-traded sector, the model fails to produce a positive cumulative response of the real wage in the baseline scenario. We believe that allowing for imperfect mobility of labor across sectors, along the lines of Horvath [2000], might solve these two puzzles. The difficulty in reallocating labor should produce a rise in sectoral capital-labor ratios because the non-traded sector experiences a small labor inflow if the cost of shifting is large enough, regardless of sectoral capital intensities. Higher sectoral capital-labor ratios lower the return of domestic capital and raises the wage rate. As a result, investment should fall while the real consumption wage should rise.

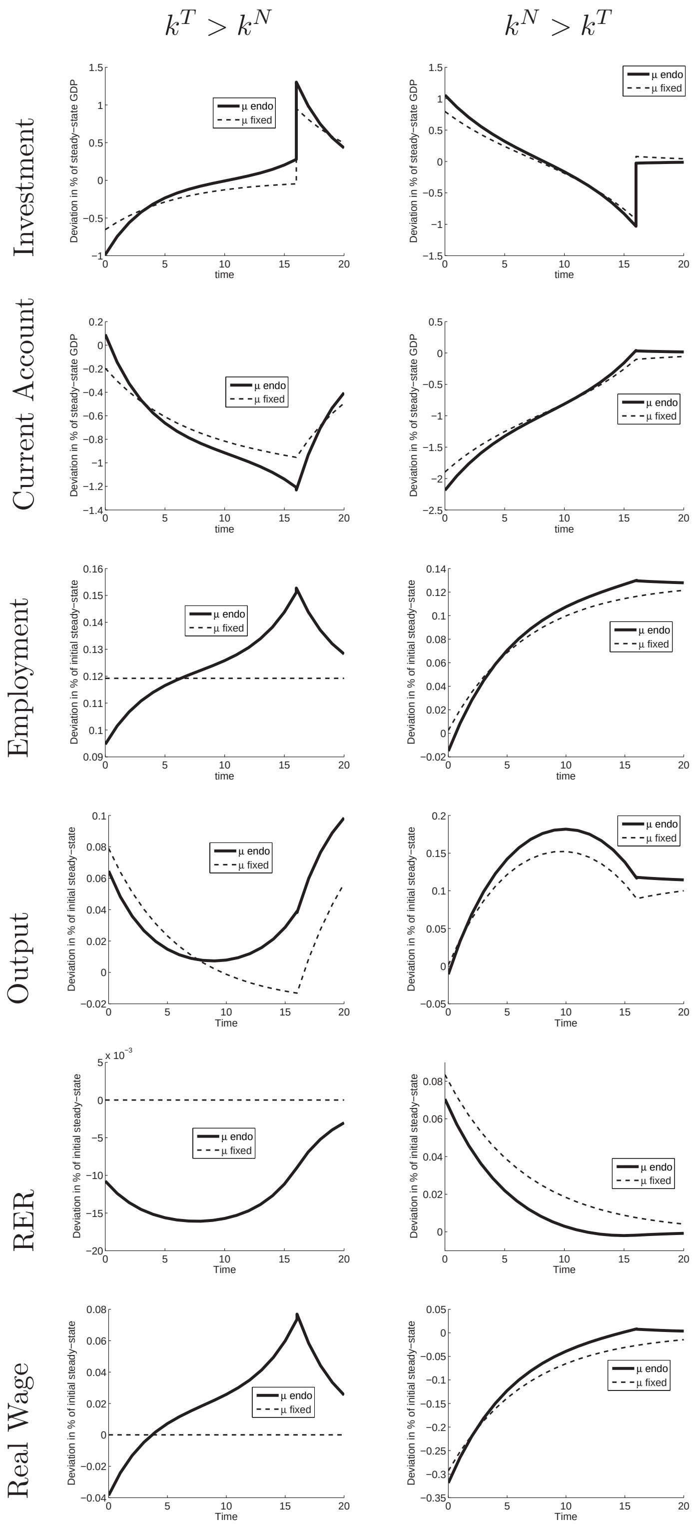


Figure 1: Effect of government spending shocks. Notes: variables are measured in percentage points of output, with the exception of employment, the real exchange rate and real consumption wage which are scaled by their initial steady-state values. The solid line shows results for an endogenous markup and the dashed line for a fixed markup.

Table 1: Data to Calibrate the Two-Sector Model (1970-2004)

Countries	Non tradable Share				G^j/Y^j		Capital Share		Markup
	Output	Labor	Consumption	Gov. spending	G^N/Y^N	G^T/Y^T	θ^T	θ^N	μ
AUT	0.65	0.60	0.44	0.90	0.28	0.07	0.28	0.32	1.42
BEL	0.67	0.65	0.44	0.85	0.30	0.09	0.33	0.35	1.34
DEU	0.64	0.61	0.44	0.91	0.30	0.06	0.22	0.33	1.45
DNK	0.70	0.67	0.43	0.93	0.40	0.07	0.32	0.32	1.40
FIN	0.58	0.57	0.44	0.84	0.34	0.09	0.27	0.30	1.32
FRA	0.69	0.64	0.40	0.93	0.33	0.06	0.22	0.35	1.35
GBR	0.62	0.66	0.52	0.93	0.33	0.05	0.30	0.28	1.37
ITA	0.63	0.56	0.36	0.91	0.29	0.06	0.42	0.39	1.60
JPN	0.64	0.61	0.39	n.a.	n.a.	n.a.	0.37	0.29	1.51
NLD	0.67	0.69	0.45	0.91	0.34	0.08	0.41	0.33	1.32
SPA	0.61	0.59	0.50	0.90	0.25	0.05	0.35	0.26	1.33
SWE	0.65	0.67	0.51	0.90	0.43	0.09	0.30	0.30	1.31
USA	0.68	0.72	0.49	0.90	0.22	0.06	0.36	0.32	1.43

Notes: Table 1 shows the non-tradable content of GDP, employment, consumption, and government spending. Table 1 also gives the share G^j/Y^j of government spending on the traded and non-traded good in the sectoral output (with $j = T, N$), the shares θ^j of capital income in output in both sectors, and the markup μ charged by the non-traded sector for 13 OECD countries. The choice of these countries has been dictated by data availability. For the countries of our sample, the period runs from 1970 to 2004. The construction and sources are detailed in a longer version of the paper which is available at `http://www.beta-umr7522.fr/productions/publications/2012/2012-17.pdf`.

Table 2: Quantitative Effects of a Temporary Fiscal Expansion (in %): The Case of an Endogenous Markup

Variables	$k^T > k^N$						$k^N > k^T$					
	μ fixed	Bench $\mathcal{T} = 16$			Short $\mathcal{T} = 8$	Long $\mathcal{T} = 32$	μ fixed	Bench $\mathcal{T} = 16$			Short $\mathcal{T} = 8$	Long $\mathcal{T} = 32$
	$\sigma_L = 0.5$	$\sigma_L = 0.5$	$\sigma_L = 0.1$	$\sigma_L = 1$	$\sigma_L = 0.5$	$\sigma_L = 0.5$	$\sigma_L = 0.5$	$\sigma_L = 0.5$	$\sigma_L = 0.1$	$\sigma_L = 1$	$\sigma_L = 0.5$	$\sigma_L = 0.5$
A.Impact												
Consumption, $dC(0)$	-0.07	-0.07	-0.12	-0.04	-0.03	-0.12	-0.08	-0.08	-0.13	-0.06	-0.04	-0.14
RER, $dP(0)$	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	0.08	0.07	0.07	0.07	0.05	0.09
Wage, $dW(0)$	0.00	-0.04	-0.05	-0.05	-0.05	-0.02	-0.25	-0.28	-0.25	-0.28	-0.22	-0.33
Real wage, $dW(0)/P_C(0)$	0.00	-0.04	-0.04	-0.05	-0.05	-0.01	-0.29	-0.32	-0.29	-0.32	-0.24	-0.38
Labor, $dL(0)$	0.12	0.09	0.04	0.09	0.03	0.21	0.00	-0.02	0.02	-0.11	-0.04	0.06
Savings, $dS(0)$	-0.85	-0.90	-0.89	-0.93	-0.98	-0.74	-1.10	-1.13	-1.04	-1.21	-1.14	-1.06
Investment, $dI(0)$	-0.66	-0.99	-1.14	-1.07	-1.22	-0.49	0.80	1.06	0.82	1.23	0.63	1.31
Current Account, $dCA(0)$	-0.20	0.09	0.25	0.14	0.24	-0.25	-1.90	-2.19	-1.86	-2.44	-1.76	-2.36
GDP, $dY(0)$	0.08	0.06	0.03	0.06	0.02	0.14	0.00	-0.01	0.01	-0.07	-0.03	0.04
Traded output, $dY^T(0)$	-0.24	0.05	0.18	0.11	0.22	-0.33	-1.92	-2.21	-1.91	-2.46	-1.78	-2.41
NT output, $dY^N(0)$	0.32	0.01	-0.16	-0.05	-0.20	0.46	1.92	2.20	1.93	2.38	1.75	2.45
B.Cumulative Response												
RER, dP	0.00	-0.11	-0.09	-0.13	-0.09	-0.13	0.42	0.29	0.28	0.26	0.22	0.35
Real wage, dW/P_C	0.00	-0.04	-0.08	-0.05	0.04	0.05	-1.47	-1.44	-1.32	-1.40	-1.06	-1.73
Investment, dI	-3.16	-3.55	-4.48	-3.27	-3.78	-1.81	3.22	4.28	3.22	5.01	-0.05	6.06
Current account, dCA	-3.91	-3.61	-2.67	-3.94	-3.87	-4.23	-11.53	-12.60	-11.15	-13.69	-8.52	-13.60
GDP, dY	0.32	0.23	-0.23	0.39	-0.09	0.97	0.69	0.78	0.58	0.68	0.22	1.41
Traded output, dY^T	-4.11	-3.84	-3.17	-4.05	-3.95	-4.70	-11.31	-12.36	-11.16	-13.31	-8.28	-13.54
NT output, dY^N	4.42	4.05	2.92	4.43	3.84	5.65	12.00	13.15	11.75	14.00	8.52	14.96

Notes: We consider a temporary rise in G^N which raises total government spending by one percentage point of GDP. Impact deviations from initial steady-state are scaled by initial GDP, exception with the real exchange rate, wage rate, labor which are scaled by their initial steady-state values. A short-lived, medium-lived and long-lived shock lasts 8, 16, 32 quarters respectively. NT means non traded.

References

Baxter, Marianne and Robert G. King (1993) Fiscal Policy in General Equilibrium. *American Economic Review*, 83(3), pp. 315-334.

Benetrix, Agustin S. (2012) Fiscal Shocks And Real Wages. *International Journal of Finance and Economics*, 17(3), pp. 203-220.

Benetrix, Agustin and Philip R. Lane (2010) Fiscal Shocks and the Sectoral Composition of Output. *Open Economies Review* 21(3), pp. 335-350.

Burnside Craig, Martin Eichenbaum, and Jonas D. M. Fisher (2004) Fiscal Shocks and their Consequences. *Journal of Economic Theory* 115, pp. 89-117.

Burstein, Ariel T., João C. Neves and Sergio Rebelo (2004) Investment Prices and Exchange Rates: Some Basic Facts. *Journal of the European Economic Association* 2(2-3), pp. 302-309.

Cardi Olivier and Gernot Müller (2011) Habit Formation and Fiscal Transmission in Open Economies. *Journal of International Economics* 85, pp. 256-267.

Cashin, Paul and John C. McDermott (2003) Intertemporal Substituion and Terms-of-Trade Shocks. *Review of International Economics*, vol. 11 (4), pp. 604-618.

Corsetti, Giancarlo, André Meier, and Gernot Müller (2012) What Determines Government Spending Multipliers? *Economic Policy*, forthcoming.

Coto-Martinez, Javier and Huw Dixon (2003) Profits, Markups and Entry: Fiscal Policy in an Open Economy. *Journal of Economic Dynamics and Control*, 27(4), pp. 573-597.

Domeij, David, and Martin Flodén (2006) The Labor-Supply Elasticity and Borrowing Constraints: Why Estimates are Biased. *Review of Economic Dynamics* 9, pp. 242-262.

Enders, Zeno, Gernot Müller and Almuth Scholl (2011) How do Fiscal and Technology Shocks affect Real Exchange Rates? New Evidence for the United States. *Journal of International Economics*, 83, pp. 53-69.

Galí, Jordi (1995) Product Diversity, Endogenous Markups, and Development Traps. *Journal of Monetary Economics*, 36, pp. 39-63.

Galí, Jordi, J. David López-Salido and J. Vallés (2007) Understanding the Effects of Government Spending on Consumption. *Journal of the European Economic Association* 5, pp. 227-270.

Hall, Robert E. (2009) By How Much Does GDP Rise If the Government Buys More Output? *Brookings Papers on Economic Activity* Fall, pp. 183-231.

Horvath, Michael (2000) Sectoral Shocks and Aggregate Fluctuations. *Journal of Monetary Economics*, 45, pp. 69-106.

Jaimovich, Nir and Joseph Floetotto (2008) Firm Dynamics, Markup Variations and the Business Cycle. *Journal of Monetary Economics*, 55, pp. 1238-1252.

Linnemann, Ludger (2001) The Price Index Effect, Entry, and Endogenous Markups in a Macroeconomic Model of Monopolistic Competition. *Journal of Macroeconomics*, 23, pp. 441-458

Monacelli, Tommaso and Robert Perotti (2010) Fiscal Policy, the Real Exchange Rate and Traded Goods. *Economic Journal* 120, pp. 437-461.

Pappa, Evi (2009) The Effects of Fiscal Shocks on Employment and the Real Wage. *International Economic Review* 50, pp. 217-244.

Perotti, Roberto (2007) In Search of the Transmission Mechanism of Fiscal Policy. *NBER Working Paper* 13143.

Ramey, Valerie A. (2011) Identifying Government Spending Shocks: It's All in the Timing *Quarterly Journal of Economics*, forthcoming.

Schubert, Stefan F., and Stephen J. Turnovsky (2002) The Dynamics of Temporary Policies in a Small Open Economy. *Review of International Economics* 10(4), pp. 604-622.

Turnovsky, Stephen J. and Partha Sen (1995) Investment in a Two-Sector Dependent Economy. *Journal of the Japanese and International Economies* 9, pp. 29-55.

Wu, Yangru, and Junxi Zhang (2000) Endogenous markups and the effects of income taxation: Theory and evidence from OECD countries. *Journal of Public Economics*, 7, pp. 383-406.

FISCAL SHOCKS IN A TWO SECTOR OPEN ECONOMY WITH ENDOGENOUS MARKUPS

TECHNICAL APPENDIX

A Short-Run Static Solutions

In this section, we compute short-run static solutions. It is worthwhile noting that in this paper, we assume that the non-traded sector is imperfectly competitive and charges a markup denoted by μ. We also allow for the markup to be endogenous in section 3 in the text. In order to isolate the influence of markup variations on variables, i.e. the competition channel, we express variables in terms of the markup; hence, we treat μ as an exogenous variable in computing short-run static solutions. For example, if a short-run static solution is given by $x = x\left(\bar{\lambda}, P, \mu\right)$ with $\bar{\lambda}$ the shadow value of wealth, P the relative price of non tradables and μ the markup, the variable x is only affected by $\bar{\lambda}$ and P in the case of fixed markup while x is influenced also by the competition channel when we allow for the markup to be endogenous. In section K, we set out the model with an imperfectly competitive non-traded sector, assuming that a limited number of competitors operate within each sector. When the number of competitors is large, the imperfectly competitive non-traded sector charges a fixed markup. In section L, we set out the model with an imperfectly competitive non-traded sector, assuming that the number of firms is fixed so that profits are no longer driven down to zero. In section M, we solve the model with endogenous markups by abstracting from physical capital and derive formal solutions for temporary fiscal shocks.

A.1 Short-Run Static Solutions for Consumption-Side

In this subsection, we compute short-run static solutions for real consumption and labor supply. Static efficiency conditions (5a) and (5b) can be solved for consumption and labor which of course must hold at any point of time:

$$C = C\left(\bar{\lambda}, P\right), \quad L = L\left(\bar{\lambda}, P, \mu\right), \tag{26}$$

with

$$C_{\bar{\lambda}} = \frac{\partial C}{\partial \bar{\lambda}} = -\sigma_C \frac{C}{\bar{\lambda}} < 0, \tag{27a}$$

$$C_P = \frac{\partial C}{\partial P} = -\alpha_C \sigma_C \frac{C}{P} < 0, \tag{27b}$$

$$L_{\bar{\lambda}} = \frac{\partial L}{\partial \bar{\lambda}} = \sigma_L \frac{L}{\bar{\lambda}} > 0, \tag{27c}$$

$$L_P = \frac{\partial L}{\partial P} = \sigma_L L \frac{W_P}{W} = -\sigma_L L \frac{1}{W} \frac{k^T h}{\mu\left(k^N - k^T\right)} \lesseqgtr 0, \tag{27d}$$

$$L_\mu = \frac{\partial L}{\partial \mu} = \sigma_L L \frac{W_\mu}{W} = \sigma_L L \frac{1}{W} \frac{k^T P h}{(\mu)^2\left(k^N - k^T\right)} \gtreqless 0, \tag{27e}$$

where σ_C and σ_L correspond to the intertemporal elasticity of substitution for consumption and labor, respectively.

Denoting by ϕ the intratemporal elasticity of substitution between the tradable and the non tradable good and inserting short-run solution for consumption (26) into intra-temporal allocations between non tradable and tradable goods, we solve for C^T and C^N:

$$C^T = C^T\left(\bar{\lambda}, P\right), \quad C^N = C^N\left(\bar{\lambda}, P\right), \tag{28}$$

with

$$
\begin{aligned}
C^T_{\bar{\lambda}} &= -\sigma_C \frac{C^T}{\bar{\lambda}} < 0, && \text{(29a)}\\
C^T_P &= \alpha_C \frac{C^T}{P} (\phi - \sigma_C) \lesseqgtr 0, && \text{(29b)}\\
C^N_{\bar{\lambda}} &= -\sigma_C \frac{C^N}{\bar{\lambda}} < 0, && \text{(29c)}\\
C^N_P &= -\frac{C^N}{P} \left[(1-\alpha_C)\phi + \alpha_C \sigma_C\right] < 0, && \text{(29d)}
\end{aligned}
$$

where we used the fact that $-\frac{P''_C P}{P'_C} = \phi(1-\alpha_C) > 0$ and $P'_C C = C^N$.

A.2 Short-Run Static Solutions for Production-Side

Capital-Labor Ratios

From static optimality conditions (8a) and (8b), we may express sector capital-labor ratios as functions of the real exchange rate:

$$
k^T = k^T(P, \mu), \qquad k^N = k^N(P, \mu), \tag{30}
$$

with

$$
\begin{aligned}
k^T_P &= \frac{\partial k^T}{\partial P} = \frac{h}{\mu f_{kk}(k^N - k^T)}, && \text{(31a)}\\
k^T_\mu &= \frac{\partial k^T}{\partial \mu} = -\frac{Ph}{(\mu)^2 f_{kk}(k^N - k^T)}, && \text{(31b)}\\
k^N_P &= \frac{\partial k^N}{\partial P} = \frac{\mu f}{P^2 h_{kk}(k^N - k^T)}. && \text{(31c)}\\
k^N_\mu &= \frac{\partial k^N}{\partial \mu} = -\frac{f}{P h_{kk}(k^N - k^T)}. && \text{(31d)}
\end{aligned}
$$

Wage

Equality $\left[f\left(k^T\right) - k^T f_k\left(k^T\right)\right] \equiv W$ can be solved for the wage rate:

$$
W = W(P, \mu), \tag{32}
$$

with

$$
\begin{aligned}
W_P &= \frac{\partial W}{\partial P} = -k^T f_{kk} k^T_P = -k^T \frac{h}{\mu(k^N - k^T)} \lesseqgtr 0, && \text{(33a)}\\
W_\mu &= -\frac{\partial W}{\partial \mu} = -k^T f_{kk} k^T_\mu = k^T \frac{Ph}{(\mu)^2 (k^N - k^T)} \gtreqless 0. && \text{(33b)}
\end{aligned}
$$

Labor

Substituting short-run static solutions for labor (26) and capital-labor ratios (30) into the resource constraints for capital and labor (9), we can solve for traded and non-traded labor as follows:

$$
L^T = L^T\left(K, P, \bar{\lambda}, \mu\right), \quad L^N = L^N\left(K, P, \bar{\lambda}, \mu\right), \tag{34}
$$

with

$$L_K^T = \frac{\partial L^T}{\partial K} = \frac{1}{k^T - k^N} \lesseqgtr 0, \tag{35a}$$

$$L_P^T = \frac{\partial L^T}{\partial P} = \frac{1}{\mu\left(k^N - k^T\right)^2}\left[\frac{L^T h}{f_{kk}} + \frac{\mu^2 L^N f}{P^2 h_{kk}} - \sigma_L L \frac{1}{W} k^T k^N h\right] < 0, \tag{35b}$$

$$L_\mu^T = \frac{\partial L^T}{\partial \mu} = -\frac{1}{\left[\mu\left(k^N - k^T\right)\right]^2}\left[\frac{L^T P h}{f_{kk}} + \frac{\mu^2 L^N f}{P h_{kk}} - \sigma_L L \frac{1}{W} k^T k^N P h\right] > 0, \tag{35c}$$

$$L_{\bar{\lambda}}^T = \frac{\partial L^T}{\partial \bar{\lambda}} = \sigma_L \frac{L}{\bar{\lambda}} \frac{k^N}{k^N - k^T} \gtreqless 0, \tag{35d}$$

$$L_K^N = \frac{\partial L^N}{\partial K} = \frac{1}{k^N - k^T} \gtreqless 0, \tag{35e}$$

$$L_P^N = \frac{\partial L^N}{\partial P} = -\frac{1}{\mu\left(k^N - k^T\right)^2}\left[\frac{L^T h}{f_{kk}} + \frac{\mu^2 L^N f}{P^2 h_{kk}} - \sigma_L L \frac{1}{W}\left(k^T\right)^2 h\right] > 0, \tag{35f}$$

$$L_\mu^N = \frac{\partial L^N}{\partial \mu} = \frac{1}{\left[\mu\left(k^N - k^T\right)\right]^2}\left[\frac{L^T P h}{f_{kk}} + \frac{\mu^2 L^N f}{P h_{kk}} - \sigma_L L \frac{1}{W}\left(k^T\right)^2 P h\right] < 0, \tag{35g}$$

$$L_{\bar{\lambda}}^N = \frac{\partial L^N}{\partial \bar{\lambda}} = -\sigma_L \frac{L}{\bar{\lambda}} \frac{k^T}{k^N - k^T} \lesseqgtr 0. \tag{35h}$$

$$\tag{35i}$$

Output

Inserting short-run static solutions for capital-labor ratios (30) and for labor (35) into the production functions, we can solve for traded output, $Y^T = L^T f\left(k^T\right)$, and non-traded output, $Y^N = L^N h\left(k^N\right)$:

$$Y^T = Y^T\left(K, P, \bar{\lambda}, \mu\right), \qquad Y^N = Y^N\left(K, P, \bar{\lambda}, \mu\right), \tag{36}$$

with

$$Y_K^T = \frac{\partial Y^T}{\partial K} = -\frac{f}{k^N - k^T} \lesseqgtr 0, \tag{37a}$$

$$Y_P^T = \frac{\partial Y^T}{\partial P} = \frac{1}{\mu\left(k^N - k^T\right)^2}\left[\frac{P L^T (h)^2}{\mu f_{kk}} + \frac{L^N (\mu f)^2}{(P)^2 h_{kk}} - \sigma_L L \frac{1}{W} k^T k^N h f\right] < 0, \tag{37b}$$

$$Y_\mu^T = \frac{\partial Y^T}{\partial \mu} = -\frac{1}{\left[\mu\left(k^N - k^T\right)\right]^2}\left[\frac{L^T (P h)^2}{\mu f_{kk}} + \frac{L^N (\mu f)^2}{P h_{kk}} - \sigma_L L \frac{1}{W} k^T k^N P h f\right] > 0, \tag{37c}$$

$$Y_{\bar{\lambda}}^T = \frac{\partial Y^T}{\partial \bar{\lambda}} = \sigma_L \frac{L}{\bar{\lambda}} \frac{k^N f}{k^N - k^T} \gtreqless 0, \tag{37d}$$

$$Y_K^N = \frac{\partial Y^N}{\partial K} = \frac{h}{k^N - k^T} \gtreqless 0, \tag{37e}$$

$$Y_P^N = \frac{\partial Y^N}{\partial P} = -\frac{1}{P\left(k^N - k^T\right)^2}\left[\frac{P L^T (h)^2}{\mu f_{kk}} + \frac{L^N (\mu f)^2}{P^2 h_{kk}} - \frac{P}{\mu} \sigma_L L \frac{1}{W}\left(k^T h\right)^2\right] > 0 \tag{37f}$$

$$Y_\mu^N = \frac{\partial Y^N}{\partial \mu} = \frac{1}{\mu\left(k^N - k^T\right)^2}\left[\frac{P L^T (h)^2}{\mu f_{kk}} + \frac{L^N (\mu f)^2}{P^2 h_{kk}} - \frac{P}{\mu} \sigma_L L \frac{1}{W}\left(k^T h\right)^2\right] < 0, \tag{37g}$$

$$Y_{\bar{\lambda}}^N = \frac{\partial Y^N}{\partial \bar{\lambda}} = -\sigma_L \frac{L}{\bar{\lambda}} \frac{k^T h}{k^N - k^T} \lesseqgtr 0, \tag{37h}$$

From (37b) and (37f), an appreciation in the real exchange rate attracts resources from the traded to the non-traded sector which in turn raises the output of the latter. From (37a) and (37e), a rise in the capital stock raises the output of the sector which is relatively more capital intensive. From (37d) and (37h), an increase in the marginal utility of wealth raises labor supply and thereby increases output in the sector which is more labor intensive.

For clarity purpose, in the text, we write out short-run static solutions by expressing output in terms of labor supply, i.e. $Y^T = Y^T(K, L, P, \mu)$ and $Y^N = Y^N(K, L, P, \mu)$. The partial derivatives of sectoral output w. r. t. to labor are:

$$Y_L^T = \frac{\partial Y^T}{\partial L} = \frac{k^N f}{k^N - k^T} \gtrless 0, \quad Y_L^N = \frac{\partial Y^N}{\partial L} = -\frac{k^T h}{k^N - k^T} \lessgtr 0. \tag{38}$$

Useful Properties

Making use of (37b) and (37f), (37a) and (37e), we deduce the following useful properties:

$$\begin{aligned}
Y_P^T + P\frac{Y_P^N}{\mu} &= -\sigma_L L \frac{k^T h}{\mu\left(k^N - k^T\right)} \lessgtr 0, && \text{(39a)}\\
Y_K^T + \frac{P}{\mu}Y_K^N &= \frac{\mu f - Ph}{\mu\left(k^T - k^N\right)} = \frac{P}{\mu}h_k = f_k, && \text{(39b)}\\
Y_L^T + P\frac{Y_L^N}{\mu} &= W, && \text{(39c)}\\
Y_\mu^T + P\frac{Y_\mu^N}{\mu} &= \sigma_L L k^T \frac{Ph}{\mu^2\left(k^N - k^T\right)} \gtrless 0, && \text{(39d)}\\
Y_{\bar{\lambda}}^T + P\frac{Y_{\bar{\lambda}}^N}{\mu} &= \sigma_L \frac{L}{\bar{\lambda}} \frac{\left(k^N \mu f - k^T P h\right)}{\mu\left(k^N - k^T\right)} = \sigma_L \frac{L}{\bar{\lambda}} W > 0, && \text{(39e)}
\end{aligned}$$

where we used the fact that $\mu f \equiv P\left[h - h_k\left(k^N - k^T\right)\right]$ and $k^N \mu f - k^T P h = P\left(h - h^K k^N\right)\left(k^N - k^T\right) = \mu W\left(k^N - k^T\right)$.

In addition, using the fact that $r^K = f_k\left[k^T(P, \mu)\right]$, the rental rate of capital denoted by r^K can be expressed as a function of the real exchange rate P and the mark-up μ:

$$r^K = r^K(P, \mu), \tag{40}$$

with partial derivatives given by:

$$\begin{aligned}
r_P^K &\equiv \frac{\partial r^K}{\partial P} = \frac{h}{\mu\left(k^N - k^T\right)} \gtrless 0, && \text{(41a)}\\
r_P^\mu &\equiv \frac{\partial r^K}{\partial \mu} = -\frac{Ph}{\mu^2\left(k^N - k^T\right)} \lessgtr 0. && \text{(41b)}
\end{aligned}$$

B Equilibrium Dynamics and Formal Solutions

Inserting short-run static solutions (26), (28) and (36) into (5d) and (12), we obtain:

$$\begin{aligned}
\dot{K} &= \frac{1}{\mu}Y^N\left(K, P, \bar{\lambda}\right) - C^N\left(\bar{\lambda}, P\right) - \delta_K K - G^N, && \text{(42a)}\\
\dot{P} &= P\left\{r^\star + \delta_K - \frac{h_k\left[(P)\right]}{\mu}\right\}. && \text{(42b)}
\end{aligned}$$

Linearizing these two equations around the steady-state, and denoting $\tilde{x} = \tilde{K}, \tilde{P}$ the long-term values of $x = K, P$, we obtain in a matrix form:

$$\left(\dot{K}, \dot{P}\right)^T = J\left(K(t) - \tilde{K}, P(t) - \tilde{P}\right)^T, \tag{43}$$

where J is given by

$$J \equiv \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \tag{44}$$

with

$$b_{11} = \frac{Y_K^N}{\mu} - \delta_K = \frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}_T\right)} - \delta_K \gtrless 0, \quad b_{12} = \frac{Y_P^N}{\mu} - C_P^N > 0, \tag{45a}$$

$$b_{21} = 0, \quad b_{22} = -\tilde{P}\frac{h_{kk}k_P^N}{\mu} = -\frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^N - \tilde{k}^T\right)} = \frac{Y_K^T}{\tilde{P}} \lesseqgtr 0. \tag{45b}$$

Equilibrium Dynamics

By denoting ν the eigenvalue of matrix J, the characteristic equation for the matrix of the linearized system (43) can be written as follows:

$$\nu^2 - \frac{1}{\tilde{P}}\left(Y_K^T + \frac{\tilde{P}}{\tilde{\mu}}Y_K^N - \delta_K\tilde{P}\right)\nu + \frac{Y_K^T}{\tilde{P}}\left(\frac{Y_K^N}{\mu} - \delta_K\right) = 0. \tag{46}$$

The determinant denoted by Det of the linearized 2×2 matrix (44) is unambiguously negative:[40]

$$\text{Det J} = b_{11}b_{22} = \frac{Y_K^T}{\tilde{P}}\left(\frac{Y_K^N}{\mu} - \delta_K\right) < 0, \tag{47}$$

and the trace denoted by Tr is given by

$$\text{Tr J} = b_{11} + b_{22} = \frac{1}{\tilde{P}}\left(Y_K^T + \frac{\tilde{P}}{\tilde{\mu}}Y_K^N\right) - -\delta_K = \frac{h_k}{\mu} - \delta_K = r^\star > 0, \tag{48}$$

where we used the fact that at the long-run equilibrium $\frac{h_k}{\mu} = r^\star + \delta_K$.

From (46), the characteristic root reads as:

$$\nu_i \equiv \frac{1}{2}\left\{r^\star \pm \sqrt{(r^\star)^2 - 4\frac{Y_K^T}{\tilde{P}}\left(\frac{Y_K^N}{\mu} - \delta_K\right)}\right\} \gtrless 0, \quad i = 1, 2. \tag{49}$$

Using (48), then (49) can be rewritten as follows:

$$\nu_i \equiv \frac{1}{2}\left\{r^\star \pm \left[\frac{Y_K^T}{\tilde{P}} - \left(\frac{Y_K^N}{\mu} - \delta_K\right)\right]\right\} \gtrless 0, \quad i = 1, 2. \tag{50}$$

We denote by $\nu_1 < 0$ and $\nu_2 > 0$ the stable and unstable real eigenvalues, satisfying

$$\nu_1 < 0 < r^\star < \nu_2. \tag{51}$$

Since the system features one state variable, K, and one jump variable, P, the equilibrium yields a unique one-dimensional stable saddle-path.

Formal Solutions

General solutions paths are given by :

$$K(t) - \tilde{K} = B_1 e^{\nu_1 t} + B_2 e^{\nu_2 t}, \tag{52a}$$

$$P(t) - \tilde{P} = \omega_2^1 B_1 e^{\nu_1 t} + \omega_2^2 B_2 e^{\nu_2 t}, \tag{52b}$$

[40] Starting with the equality of labor marginal products across sectors, using the fact that $f_k = \frac{P}{\mu}h_k$ and $h_k/\mu = r^\star + \delta_K$, it is straightforward to prove that b_{11} is positive in the case $k^N > k^T$.

where we normalized ω_1^i to unity. The eigenvector ω_2^i associated with eigenvalue μ_i is given by

$$\omega_2^i = \frac{\nu_i - b_{11}}{b_{12}}, \tag{53}$$

with

$$b_{11} = \frac{Y_K^N}{\mu} - \delta_K = \frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}_T\right)} - \delta_K \gtrless 0, \tag{54a}$$

$$b_{12} = \frac{Y_P^N}{\mu} - C_P^N > 0, \tag{54b}$$

where C_P^N is given by (29d).

Case $k^N > k^T$

This assumption reflects the fact that the capital-labor ratio of the non-traded good sector exceeds the capital-labor of the traded sector. From (51), the stable and unstable eigenvalues can be rewritten as follows:

$$\nu_1 = -\frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^N - \tilde{k}^T\right)} < 0, \tag{55a}$$

$$\nu_2 = \frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}^T\right)} - \delta_K > 0, \tag{55b}$$

since we suppose that $k^N > k^T$.

We can deduce the signs of several useful expressions:

$$Y_K^N = \mu\left(\nu_2 + \delta_K\right) > 0, \tag{56a}$$

$$Y_K^T = \tilde{P}\nu_1 < 0, \tag{56b}$$

$$\frac{\tilde{P}h_{kk}k_P^N}{\mu} = -\nu_1 > 0, \tag{56c}$$

$$Y_{\bar{\lambda}}^N = -\frac{1}{\bar{\lambda}}\sigma_L\tilde{L}\tilde{k}^T\mu\left(\nu_2 + \delta_K\right) < 0, \tag{56d}$$

$$Y_{\bar{\lambda}}^T = -\frac{1}{\bar{\lambda}}\sigma_L\tilde{L}\tilde{P}\tilde{k}^N\nu_1 > 0. \tag{56e}$$

We write out eigenvector ω^i associated with eigenvalue ν_i (with $i = 1, 2$), to determine their signs:

$$\omega^1 = \begin{pmatrix} 1 & (+) \\ \frac{\nu_1 - \nu_2}{\left(\frac{Y_P^N}{\mu} - C_P^N\right)} & (-) \end{pmatrix}, \quad \omega^2 = \begin{pmatrix} 1 & (+) \\ 0 & \end{pmatrix}. \tag{57}$$

Case $k^T > k^N$

This assumption reflects the fact that the capital-labor ratio of the traded good sector exceeds the capital-labor ratio of the non-traded sector. From (51), the stable and unstable eigenvalues can be rewritten as follows:

$$\nu_1 = \frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}^T\right)} - \delta_K < 0, \tag{58a}$$

$$\nu_2 = -\frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^N - \tilde{k}^T\right)} > 0, \tag{58b}$$

since we suppose that $k^T > k^N$.

We can deduce the signs of several useful expressions:

$$\begin{aligned} Y_K^N &= \mu\left(\nu_1+\delta_K\right) < 0, && (59a)\\ Y_K^T &= \tilde{P}\nu_2 > 0, && (59b)\\ \frac{\tilde{P}h_{kk}k_P^N}{\mu} &= -\nu_2 < 0, && (59c)\\ Y_{\bar{\lambda}}^N &= -\frac{1}{\bar{\lambda}}\sigma_L\tilde{L}\tilde{k}^T\mu\left(\nu_1+\delta_K\right) > 0, && (59d)\\ Y_{\bar{\lambda}}^T &= -\frac{1}{\bar{\lambda}}\sigma_L\tilde{L}\tilde{P}\tilde{k}^N\nu_2 < 0. && (59e)\end{aligned}$$

We write out eigenvector ω^i associated with eigenvalue ν_i (with $i = 1, 2$), to determine their signs:

$$\omega^1 = \begin{pmatrix} 1 & (+) \\ 0 & \end{pmatrix}, \quad \omega^2 = \begin{pmatrix} 0 & \\ \frac{\nu_2-\nu_1}{\left(\frac{Y_P^N}{\mu}-C_P^N\right)} & (+) \end{pmatrix}. \qquad (60)$$

Formal Solution for the Stock of Foreign Assets

We first linearize equation (24) around the steady-state:

$$\dot{B}(t) = r^{\star}\left(B(t)-\tilde{B}\right) + Y_K^T\left(K(t)-\tilde{K}\right) + \left[Y_P^T - C_P^T\right]\left(P(t)-\tilde{P}\right). \qquad (61)$$

where C_P^T is given by (29b).

Inserting general solutions for $K(t)$ and $P(t)$, the solution for the stock of international assets is given by follows:

$$\dot{B}(t) = r^{\star}\left(B(t)-\tilde{B}\right) + Y_K^T\sum_{i=1}^{2}B_i e^{\nu_i t} + \left[Y_P^T - C_P^T\right]\sum_{i=1}^{2}B_i\omega_2^i e^{\nu_i t}. \qquad (62)$$

Solving the differential equation leads to the following expression:

$$B(t)-\tilde{B} = \left[\left(B_0-\tilde{B}\right) - \Phi_1 B_1 - \Phi_2 B_2\right]e^{r^{\star}t} + \Phi_1 B_1 e^{\nu_1 t} + \Phi_2 B_2 e^{\nu_2 t}, \qquad (63)$$

with

$$\Phi_i = \frac{N_i}{\nu_i - r^{\star}} = \frac{Y_K^T + \left[Y_P^T - C_P^T\right]\omega_2^i}{\nu_i - r^{\star}}, \quad i = 1, 2. \qquad (64)$$

Invoking the transversality condition for intertemporal solvency, the terms in brackets of equation (53) must be null and we must set $B_2 = 0$. We obtain the linearized version of the nation's intertemporal budget constraint:

$$B_0 - \tilde{B} = \Phi_1\left(K_0 - \tilde{K}\right). \qquad (65)$$

The stable solution for net foreign assets finally reduces to:

$$B(t) - \tilde{B} = \Phi_1\left(K(t) - \tilde{K}\right). \qquad (66)$$

Case $k^N > k^T$

$$\begin{aligned} N_1 &= Y_K^T + \left(Y_P^T - C_P^T\right)\omega_2^1, \\ &= \tilde{P}\nu_2\left\{1 + \frac{\omega_2^1}{\tilde{P}\nu_2}\left[\sigma_C\tilde{C}^N - \sigma_L\tilde{L}\tilde{k}^T\left(\nu_2+\delta_K\right)\right]\right\} \gtrless 0, && (67a)\\ N_2 &= Y_K^T + \left(Y_P^T - C_P^T\right)\omega_2^2, && (67b)\\ &= Y_K^T = \tilde{P}\nu_1 < 0, && (67c)\end{aligned}$$

where (67c) follows from the fact that $\omega_2^2 = 0$. We made use of property (338) together with the fact that $C_P^T = P_C C_P - P C_P^N$ to compute $Y_P^T - C_P^T = -\tilde{P}\left(\frac{Y_P^N}{\mu} - C_P^N\right) - P_C C_P - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right) \gtrless 0$.

The sign of Φ_1 is ambiguous and reflects the impact of capital accumulation on the foreign asset accumulation along a stable transitional path:

$$\dot{B}(t) = \Phi_1 \dot{K}(t).$$

where $\dot{K}(t) = \nu_1 B_1 e^{\nu_1 t}$. Following empirical evidence suggesting that the current account and investment are negatively correlated (see e. g. Glick and Rogoff [1995]), we will impose thereafter:

Assumption 1 $\Phi_1 < 0$ *which implies that* $N_1 > 0$.

The condition for the assumption to hold, i. e. $N_1 > 0$, may be rewritten as follows:

$$\nu_2 > -\frac{\omega_2^1}{\tilde{P}}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\right]. \tag{68}$$

Note that, for all parametrization, we find $\Phi_1 < 0$. Using (64), Φ_i $(i = 1, 2)$ can be written as follows:

$$\Phi_1 = -\tilde{P}\left\{1 + \frac{\omega_2^1}{\tilde{P}\nu_2}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\right]\right\} < 0, \quad \Phi_2 = -\tilde{P} < 0. \tag{69}$$

Case $k^T > k^N$

$$\begin{aligned} N_1 &= Y_K^T + \left(Y_P^T - C_P^T\right)\omega_2^1, \\ &= Y_K^T = \tilde{P}\nu_2 > 0, && (70a) \\ N_2 &= Y_K^T + \left(Y_P^T - C_P^T\right)\omega_2^2, \\ &= \tilde{P}\nu_1\left\{1 + \frac{\omega_2^2}{\tilde{P}\nu_1}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_1 + \delta_k\right)\right]\right\}, \lesseqgtr 0, && (70b) \end{aligned}$$

where (70b) follows from the fact that $\omega_2^1 = 0$. We made use of property (338) together with $C_P^T = P_C C_P - P C_P^N$ to compute $Y_P^T - C_P^T = -\tilde{P}\left(\frac{Y_P^N}{\mu} - C_P^N\right) - P_C C_P - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_1 + \delta_K\right) \gtrless 0$.

Using (64), Φ_i $(i = 1, 2)$ can be written as follows:

$$\Phi_1 = -\tilde{P} < 0, \quad \Phi_2 = -\tilde{P}\left\{1 + \frac{\omega_2^2}{\tilde{P}\nu_1}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_1 + \delta_k\right)\right]\right\} < 0. \tag{71}$$

C Derivation of the Current Account Equation

In this section, we derive the current account equation. Substituting the definition of lump-sum taxes Z by using (10), the market clearing condition for non-traded goods (12) into (3) we get:

$$\begin{aligned} \dot{B} &= r^\star B + r^K K(t) + WL - P_C C - PI - Z, \\ &= r^\star B + \left(r^K K + WL\right) - P_C C - P\left(\frac{Y^N}{\mu} - C^N - G^N\right). \end{aligned}$$

Using the fact that $L^T + L^N = L$, $K^T + K^N = K$, , the dynamic equation for the current account can be rewritten as follows:

$$\begin{aligned} \dot{B} &= r^\star B + \left[WL^T + r^K K^T\right] + \left[WL^N + r^K K^N\right] - P\frac{Y^N}{\mu} - C^T - G^T, \\ &= r^\star B + Y^T - C^T - G^T, \end{aligned}$$

where variable cost $WL^N + r^K K^N$ in the non-traded sector and output net of fixed cost in that sector, i. e. $\frac{Y^N}{\mu} = Z^N$, cancel each other.[41]

D Long-Run Effects of Permanent Fiscal Shocks: The Case of Elastic Labor Supply

In this section, we derive the steady-state effects of permanent fiscal shocks by maintaining the assumption of an elastic labor supply. Since we assume free entry, then we set $\tilde{\Pi}^N = 0$ into eq. (17).

Inserting first the appropriate short-un static solutions, the steady-state of the economy is obtained by setting $\dot{K}, \dot{P}, \dot{B} = 0$ and is defined by the following set of equations:

$$\frac{h_k\left[k^N\left(\tilde{P}\right)\right]}{\mu} = r^\star + \delta_K, \tag{72a}$$

$$\frac{Y^N\left(\tilde{K}, \tilde{P}, \bar{\lambda}\right)}{\mu} - C^N\left(\bar{\lambda}, \tilde{P}\right) - G^N - \delta_K \tilde{K} = 0, \tag{72b}$$

$$r^\star \tilde{B} + Y^T\left(\tilde{K}, \tilde{P}, \bar{\lambda}\right) - C^T\left(\bar{\lambda}, \tilde{P}\right) - G^T = 0, \tag{72c}$$

and the intertemporal solvency condition

$$\left(B_0 - \tilde{B}\right) = \Phi\left(K_0 - \tilde{K}\right). \tag{72d}$$

The steady-state equilibrium composed by these four equations jointly determine $\tilde{P}$, $\tilde{K}$, $\tilde{B}$ and $\bar{\lambda}$.

We totally differentiate the system (72) evaluated at the steady-state which yields in a matrix form:

$$\left(\begin{array}{cccc} \frac{h_{kk}k_P^N}{\mu} & 0 & 0 & 0 \\ \left(\frac{Y_P^N}{\mu} - C_P^N\right) & \frac{Y_K^N}{\mu} - \delta_K & \left(\frac{Y_{\bar{\lambda}}^N}{\mu} - C_{\bar{\lambda}}^N\right) & 0 \\ \left(Y_P^T - C_P^T\right) & Y_K^T & \left(Y_{\bar{\lambda}}^T - C_{\bar{\lambda}}^T\right) & r^\star \\ 0 & -\Phi_1 & 0 & 1 \end{array}\right)\left(\begin{array}{c} \mathrm{d}\tilde{P} \\ \mathrm{d}\tilde{K} \\ \mathrm{d}\bar{\lambda} \\ \mathrm{d}\tilde{B} \end{array}\right) = \left(\begin{array}{c} 0 \\ \mathrm{d}G^N \\ \mathrm{d}G^T \\ 0 \end{array}\right) \tag{73}$$

The determinant denoted by D of the matrix of coefficients is given by:

$$D \equiv \frac{h_{kk}k_P^N}{\mu}\left\{\left(\frac{Y_K^N}{\mu} - \delta_K\right)\left(Y_{\bar{\lambda}}^T - C_{\bar{\lambda}}^T\right) - \left(\frac{Y_{\bar{\lambda}}^N}{\mu} - C_{\bar{\lambda}}^N\right)\left[Y_K^T + r^\star \Phi_1\right]\right\} \tag{74}$$

We have to consider two cases, depending on wether the non-traded sector is more or less capital intensive than the traded sector:

$$D = -\frac{\nu_1\nu_2}{\tilde{P}\bar{\lambda}}\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right) > 0, \quad \text{if} \quad k^T > k^N, \tag{75a}$$

$$D = -\frac{\nu_1\nu_2}{\tilde{P}\bar{\lambda}}\left\{\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right) + \frac{r^\star}{\nu_2}\frac{\omega_2^1}{\nu_2}\left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\right)^2\right\} > 0, \quad \text{if} \quad k^N > k^T, \tag{75b}$$

where we used the fact that and $fk^N - Phk^T = W\left(k^N - k^T\right)$ together with $-P\left[k^N\nu_2 + k^T\left(\nu_1 + \delta_K\right)\right] \equiv W$ if $k^T > k^N$ or $-P\left[k^N\nu_1 + k^T\left(\nu_2 + \delta_K\right)\right] \equiv W$ if $k^N > k^T$.

[41] In the traded sector which is perfectly competitive, we have : $Y^T = F_L L^T + r^K K^T = WL^T + r^K K^T$. Instead, in the non-traded sector which is imperfectly competitive we have: $PZ^N = P\frac{H_L}{\mu}L^N + P\frac{H_K}{\mu}K^N$ or $P\mu Z^N = PY^N = PH_L L^N + PH_K K^N = WL^N + r^K K^N$.

D.1 A Permanent Rise in G^T

Case $k^N > k^T$

If $k^N > k^T$, the steady-state changes after a permanent rise in G^T are:

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T} = \frac{\sigma_C \tilde{C}}{\tilde{P}\bar{\lambda}} \frac{\nu_1 \nu_2}{D} < 0, \tag{76a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T} = -\frac{\nu_1 \nu_2}{\tilde{P} D} > 0, \tag{76b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^T} = 0, \tag{76c}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T} = \frac{\nu_1}{\tilde{P}\bar{\lambda}D} \left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \nu_2\right) \lesseqgtr 0, \tag{76d}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T} = \Phi_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T} \gtrless 0. \tag{76e}$$

Case $k^T > k^N$

If $k^T > k^N$, the steady-state changes after a permanent rise in G^T are:

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T} = -\frac{\sigma_C \tilde{C}}{\left(\sigma_L \tilde{W} \tilde{L} + \sigma_C P_C \tilde{C}\right)} < 0, \tag{77a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T} = \frac{\bar{\lambda}}{\left(\sigma_L \tilde{W} \tilde{L} + \sigma_C P_C \tilde{C}\right)} > 0, \tag{77b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^T} = 0, \tag{77c}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T} = \frac{\left(\sigma_L \tilde{L} \tilde{k}^T \left(\nu_1 + \delta_K\right) - \sigma_C \tilde{C}^N\right)}{\nu_1 \left(\sigma_L \tilde{W} \tilde{L} + \sigma_C P_C \tilde{C}\right)} > 0, \tag{77d}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T} = -\frac{\tilde{P}\left(\sigma_L \tilde{L} \tilde{k}^T \left(\nu_1 + \delta_K\right) - \sigma_C \tilde{C}^N\right)}{\nu_1 \left(\sigma_L \tilde{W} \tilde{L} + \sigma_C P_C \tilde{C}\right)} < 0. \tag{77e}$$

D.2 A Permanent Rise in G^N

Case $k^N > k^T$

If $k^N > k^T$, the steady-state changes after a permanent rise in G^N are:

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N} = \frac{\sigma_C \tilde{C}}{\bar{\lambda}} \frac{\nu_1 \nu_2}{D} \left[1 + \frac{r^\star}{\nu_2} \frac{\omega_2^1}{\tilde{P}\nu_2} \left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \left(\nu_2 + \delta_K\right)\right)\right] < 0, \tag{78a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = -\frac{\nu_1 \nu_2}{D} \left[1 + \frac{r^\star}{\nu_2} \frac{\omega_2^1}{\tilde{P}\nu_2} \left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \left(\nu_2 + \delta_K\right)\right)\right] > 0, \tag{78b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^N} = 0, \tag{78c}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = \frac{\nu_1}{\bar{\lambda} D \tilde{P}} \left(\sigma_L \tilde{L} \tilde{k}^N \tilde{P} \nu_1 - \sigma_C \tilde{C}^T\right) > 0, \tag{78d}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N} = -\frac{\nu_1}{\bar{\lambda} D \tilde{P}} \left(\sigma_L \tilde{L} \tilde{k}^N \tilde{P} \nu_1 - \sigma_C \tilde{C}^T\right) \left\{1 + \frac{\omega_2^1}{\tilde{P}\nu_2} \left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \left(\nu_2 + \delta_K\right)\right]\right\} < 0. \tag{78e}$$

Case $k^T > k^N$

If $k^T > k^N$, the steady-state changes after a permanent rise in G^N are:

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N} = -\frac{\sigma_C \tilde{C}\tilde{P}}{\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right)} < 0, \tag{79a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = \frac{\bar{\lambda}\tilde{P}}{\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right)} > 0, \tag{79b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^N} = 0, \tag{79c}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = -\frac{\left(\sigma_L \tilde{L}\tilde{P}\tilde{k}^N \nu_2 - \sigma_C \tilde{C}^T\right)}{\nu_1 \left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right)} \lesseqgtr 0, \tag{79d}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N} = \frac{\tilde{P}\left(\sigma_L \tilde{L}\tilde{P}\tilde{k}^N \nu_2 - \sigma_C \tilde{C}^T\right)}{\nu_1 \left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right)} \gtreqless 0. \tag{79e}$$

D.3 Rewriting the Long-Run Effects

In this subsection, we rewrite expressions of steady-state changes (78) following a permanent fiscal expansion, i.e. after a rise in G^N, when $k^N > k^T$. To begin with, it is useful to introduce some notations:

$$\tilde{\Psi} = \left[\sigma_L \tilde{L}\tilde{k}^T \left(\nu_2 + \delta_K\right) - \sigma_C \tilde{C}^N\right] \gtreqless 0, \tag{80}$$

where $\tilde{\Psi} > 0$ if labor supply is elastic enough.

$k^N > k^T$

Using notation (80), determinant D given by (75b) can be rewritten as follows:

$$D \equiv -\frac{\nu_1 \nu_2}{\tilde{P}\bar{\lambda}} \left[\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right) + \frac{r^\star \omega_2^1}{\left(\nu_2\right)^2} \tilde{\Psi}^2\right] > 0. \tag{81}$$

If $k^N > k^T$, the steady-state changes after a permanent rise in G^N are:

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = -\frac{\bar{\lambda}\left[\tilde{P} - \frac{r^\star \omega_2^1}{\left(\nu_2\right)^2}\tilde{\Psi}\right]}{\left[\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right) + \frac{r^\star \omega_2^1}{\left(\nu_2\right)^2}\tilde{\Psi}^2\right]} > 0, \tag{82a}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = -\frac{\left(\sigma_L \tilde{L}\tilde{k}^N \tilde{P}\nu_1 - \sigma_C \tilde{C}^T\right)}{\nu_2 \left[\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right) + \frac{r^\star \omega_2^1}{\left(\nu_2\right)^2}\tilde{\Psi}^2\right]} > 0, \tag{82b}$$

D.4 Impact Effects

This section estimates the impact effects of a permanent fiscal expansion. The stable adjustment of the economy is described by a saddle-path in (K, P)-space. The capital stock, the real exchange rate, and the stock of traded bonds evolve according to:

$$K(t) = \tilde{K} + B_1 e^{\nu_1 t}, \tag{83a}$$

$$P(t) = \tilde{P} + \omega_2^1 B_1 e^{\nu_1 t}, \tag{83b}$$

$$B(t) = \tilde{B} + \Phi_1 B_1 e^{\nu_1 t}, \tag{83c}$$

where $\omega_2^1 = 0$, $\Phi_1 = -\tilde{P}$ if $k^T > k^N$ and with

$$B_1 = K_0 - \tilde{K} = -\mathrm{d}\tilde{K},$$

where we used the fact that K is initially predetermined, i.e., $K(0) = K_0$.

We derive below the initial reactions of investment and the current account.

$k^N > k^T$

Differentiating (364a) w.r.t. time, evaluating at time $t = 0$, and substituting (82b), the initial response of investment is:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{perm} = -\nu_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = \frac{\nu_1 \left(\sigma_L \tilde{L}\tilde{P}\tilde{k}^T \nu_1 - \sigma_C \tilde{C}^T\right)}{\nu_2 \left[\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right) + \frac{r^\star \omega_2^1}{(\nu_2)^2}\left(\tilde{\Psi}\right)^2\right]} > 0. \tag{84}$$

Using the fact that $\Phi_1 = -\tilde{P}$, the initial reaction of the current account is:

$$\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{perm} = -\Phi_1 \nu_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = -\left(\tilde{P} - \frac{\omega_2^1}{\nu_2}\tilde{\Psi}\right) \left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{perm},$$

where we used the notation $\tilde{\Psi}$ given by eq. (80) to rewrite Φ_1 given by (69).

$k^T > k^N$

Differentiating (364a) w.r.t. time, evaluating at time $t = 0$, and substituting (79d), the initial response of investment is:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{perm} = -\nu_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = \frac{\left(\sigma_L \tilde{L}\tilde{P}\tilde{k}^N \nu_2 - \sigma_C \tilde{C}^T\right)}{\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right)} \gtrless 0. \tag{85}$$

Using the fact that $\Phi_1 = -\tilde{P}$, the initial reaction of the current account is:

$$\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{perm} = \tilde{P}\nu_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = -\tilde{P}\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{perm} \lessgtr 0.$$

E Long-Run Effects of Permanent Fiscal Shocks: The Case of Inelastic Labor Supply

In this section, we derive the steady-state effects of permanent fiscal shocks by assuming that labor supply is inelastically supplied.

We have to consider two cases, depending on whether the non-traded sector is more or less capital intensive than the traded sector :

$$D = -\frac{\nu_1 \nu_2 P_C \tilde{C} \sigma_C}{\tilde{P}\bar{\lambda}} > 0, \text{ if } k^T > k^N, \tag{86a}$$

$$D = -\frac{\nu_1 P_C \tilde{C} \sigma_C}{\tilde{P}\bar{\lambda}} \left[\nu_2 + \alpha_c \frac{r^\star}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right] > 0, \text{ if } k^N > k^T. \tag{86b}$$

The term in square brackets on the right-hand side of (86b) is positive if the following inequality holds

$$\nu_2 > -\alpha_C \frac{r^\star}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1. \tag{87}$$

From (35f), this inequality is satisfied since $\alpha_C \dfrac{r^\star}{\nu_2} < 1$.

E.1 Long-Run Effects of a Rise in G^T

Case $k^N > k^T$

$$
\begin{aligned}
\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T} &= -\frac{1}{\tilde{P}_c\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} < 0, && (88a)\\
\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T} &= \frac{\alpha_C\bar{\lambda}}{\sigma_C\tilde{P}\tilde{C}^N\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} > 0, && (88b)\\
\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^T} &= 0, && (88c)\\
\frac{\mathrm{d}\tilde{L^T}}{\mathrm{d}G^T} &= \frac{\alpha_C}{\tilde{P}\tilde{h}\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} > 0, && (88d)\\
\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T} &= -\frac{\alpha_C}{\tilde{P}\nu_2\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} < 0, && (88e)\\
\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T} &= \frac{\alpha_C}{\nu_2}\frac{\left[1+\frac{1}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]}{\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} > 0. && (88f)
\end{aligned}
$$

Case $k^T > k^N$

$$
\begin{aligned}
\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T} &= -\frac{1}{\tilde{P}_c} < 0, && (89a)\\
\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T} &= \frac{\alpha_C\bar{\lambda}}{\sigma_C\tilde{P}\tilde{C}^N} > 0, && (89b)\\
\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^T} &= 0, && (89c)\\
\frac{\mathrm{d}\tilde{L^T}}{\mathrm{d}G^T} &= \frac{\alpha_C}{\tilde{P}\tilde{h}} > 0, && (89d)\\
\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T} &= -\frac{\alpha_C}{\tilde{P}\nu_1} > 0, && (89e)\\
\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T} &= \frac{\alpha_C}{\nu_1} < 0. && (89f)
\end{aligned}
$$

E.2 Long-Run Effects of a Rise in G^N

Case $k^N > k^T$

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N} = -\frac{\tilde{P}}{\tilde{P}_C}\frac{\left[1+\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]}{\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} < 0, \tag{90a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = \frac{\alpha_c\bar{\lambda}}{\sigma_C\tilde{C}^N}\frac{\left[1+\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]}{\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} > 0, \tag{90b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^N} = 0, \tag{90c}$$

$$\frac{\mathrm{d}\tilde{L^T}}{\mathrm{d}G^N} = -\frac{(1-\alpha_C)}{\tilde{h}\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} < 0, \tag{90d}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = \frac{(1-\alpha_C)}{\nu_2\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} > 0, \tag{90e}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N} = -\frac{\tilde{P}(1-\alpha_C)\left[1+\frac{1}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]}{\nu_2\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} < 0. \tag{90f}$$

Case $k^T > k^N$

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N} = -\frac{\tilde{P}}{\tilde{P}_C} < 0, \tag{91a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = \frac{\alpha_c\bar{\lambda}}{\sigma_C\tilde{P}\tilde{C}^N} > 0, \tag{91b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^N} = 0, \tag{91c}$$

$$\frac{\mathrm{d}\tilde{L^T}}{\mathrm{d}G^N} = -\frac{(1-\alpha_C)}{\tilde{h}} < 0, \tag{91d}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = \frac{(1-\alpha_C)}{\nu_1} < 0, \tag{91e}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N} = -\frac{\tilde{P}(1-\alpha_C)}{\nu_1} > 0. \tag{91f}$$

F Derivation of Formal Solutions after Temporary Fiscal Shocks with Inelastic Labor Supply

In this section, we provide the main steps to derive formal solutions for key variables after temporary fiscal shocks, by applying the procedure developed by Schubert and Turnovsky [2002]. For simplicity purpose, we assume that $\mu = 1$ and $\delta_K = 0$ since our objective is to derive transitional dynamics analytically.

F.1 Steady-State

As in Schubert and Turnovsky [2002], we define a viable steady-state i starting at time $\mathcal{T}_i$ to be one that is consistent with long run solvency, given the stocks of capital, $K_{\mathcal{T}_i}$ and foreign bonds, $B_{\mathcal{T}_i}$. We rewrite the system of steady-state equations for an arbitrary period

i (with $i = 0, 1, 2$):

$$h_k \left[\tilde{k}^N \left(\tilde{P}_i \right) \right] = r^\star, \tag{92a}$$

$$Y^N \left(\tilde{K}_i, \tilde{P}_i \right) - \tilde{C}_i^N - G_i^N = 0, \tag{92b}$$

$$r^\star \tilde{B}_i + Y^T \left(\tilde{K}_i, \tilde{P}_i \right) - \tilde{C}_i^T - G_i^T = 0, \tag{92c}$$

together with the intertemporal solvency condition

$$\left(\tilde{B}_i - B_{\mathcal{T}_i} \right) = \Phi_1 \left(\tilde{K}_i - K_{\mathcal{T}_i} \right). \tag{92d}$$

F.2 Steady-State Functions

The new consistent procedure consists in two steps. In a **first step**, we solve the system (92a)-(92c) for $\tilde{P}_i$, $\tilde{K}_i$ and $\tilde{B}_i$ as functions of the marginal utility of wealth, $\bar{\lambda}_i$, the government expenditure on the traded and non-traded goods, i.e. G^T and G^N. Totally differentiating equations (92a)-(92c) yields in matrix form:

$$\begin{pmatrix} h_{kk} k_P^N & 0 & 0 \\ \left(Y_P^N - C_P^N \right) & Y_K^N & 0 \\ \left(Y_P^T - C_P^T \right) & Y_K^T & r^\star \end{pmatrix} \begin{pmatrix} \mathrm{d}\tilde{P}_i \\ \mathrm{d}\tilde{K}_i \\ \mathrm{d}\tilde{B}_i \end{pmatrix} = \begin{pmatrix} 0 \\ P_C' C_{\bar{\lambda}} \mathrm{d}\bar{\lambda}_i + \mathrm{d}G_i^N \\ \left(1 - \alpha_C \right) P_C C_{\bar{\lambda}} \mathrm{d}\bar{\lambda}_i + \mathrm{d}G_i^T \end{pmatrix} \tag{93}$$

The equilibrium value of the marginal utility of wealth $\bar{\lambda}_i$ and fiscal policy parameters, G_i^T, G_i^N, determine the following steady-state values:

$$\tilde{P}_i = \text{constant}, \tag{94a}$$

$$\tilde{K}_i = K \left(\bar{\lambda}_i, G_i^N \right), \tag{94b}$$

$$\tilde{B}_i = B \left(\bar{\lambda}_i, G_i^T, G_i^N \right), \tag{94c}$$

with partial derivatives given by:

$$K_{\bar{\lambda}} \equiv \frac{\partial \tilde{K}_i}{\partial \bar{\lambda}_i} = \frac{h_{kk} k_P^N P_C P_C' r^\star}{G} = -\sigma_C \frac{\tilde{C}_i^N}{\bar{\lambda}_i} \frac{\left(\tilde{k}_i^N - \tilde{k}_i^T \right)}{\tilde{h}_i} \lesseqgtr 0, \tag{95a}$$

$$\begin{aligned} B_{\bar{\lambda}} \equiv \frac{\partial \tilde{B}_i}{\partial \bar{\lambda}_i} &= \frac{h_{kk} k_P^N P_C \left(-P_C' Y_K^T + \left(1 - \alpha_C \right) P_C Y_K^N \right)}{G}, \\ &= \frac{P_C \tilde{C}_i}{\bar{\lambda}_i} \frac{\sigma_C}{Y_K^N r^\star} \left[\alpha_C r^\star - \frac{\tilde{h}_i}{\left(\tilde{k}_i^N - \tilde{k}_i^T \right)} \right], \\ &= -\frac{P_C \tilde{C}_i}{\bar{\lambda}_i} \frac{\sigma_C}{r^\star \tilde{P} \tilde{h}_i} \left[\alpha_C \tilde{f}_i + \left(1 - \alpha_C \right) \tilde{P}_i \tilde{h}_i \right] < 0, \end{aligned} \tag{95b}$$

and

$$K_{G^T} \equiv \frac{\partial \tilde{K}_i}{\partial G_i^T} = 0, \tag{96a}$$

$$B_{G^T} \equiv \frac{\partial \tilde{B}_i}{\partial G_i^T} = \frac{1}{r^\star} > 0, \tag{96b}$$

and

$$K_{G^N} \equiv \frac{\partial \tilde{K}_i}{\partial G_i^N} = \frac{h_{kk} k_P^N u_{cc} r^\star}{G} = \frac{\left(\tilde{k}_i^N - \tilde{k}_i^T \right)}{\tilde{h}_i} \gtreqless 0, \tag{97a}$$

$$B_{G^N} \equiv \frac{\partial \tilde{B}_i}{\partial G_i^N} = -\frac{h_{kk} k_P^N u_{cc} Y_K^T}{G} = \frac{\tilde{f}_i}{\tilde{h}_i} \frac{1}{r^\star} > 0, \tag{97b}$$

where $G \equiv h_{kk} k_P^N u_{cc} Y_K^N r^\star$ which simplifies as follows :

$$G \equiv \frac{\tilde{f}\tilde{h}}{\tilde{P}^2 \left(\tilde{k}^N - \tilde{k}^T\right)^2} u_{cc} r^\star < 0. \tag{98}$$

The **second step** consists to determine the equilibrium change of $\bar{\lambda}_i$ by taking the total differential of the intertemporal solvency condition (92d):

$$\left[B_{\bar{\lambda}} - \Phi_1 K_\lambda\right] \mathrm{d}\bar{\lambda}_i = \mathrm{d}B_{\mathcal{T}_i} - \Phi_1 \mathrm{d}K_{\mathcal{T}_i} - \left[B_{G^N} - \Phi_1 K_{G^N}\right] \mathrm{d}G_i^N - B_{G^T} \mathrm{d}G_i^T, \tag{99}$$

from which may solve for the equilibrium value of $\bar{\lambda}_i$ as a function of initial stocks at time $\mathcal{T}_i$ and government spending:

$$\bar{\lambda} = \lambda \left(K_{\mathcal{T}_i}, B_{\mathcal{T}_i}, G^T, G^N\right), \tag{100}$$

with

$$\begin{aligned}
\lambda_K \equiv \frac{\partial \bar{\lambda}_i}{\partial K_{\mathcal{T}_i}} &= -\frac{\Phi_1}{\left[B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right]} < 0, && (101a)\\
\lambda_B \equiv \frac{\partial \bar{\lambda}_i}{\partial B_{\mathcal{T}_i}} &= \frac{1}{\left[B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right]} < 0, && (101b)\\
\lambda_{G^T} \equiv \frac{\partial \bar{\lambda}_i}{\partial G_i^T} &= -\frac{B_{G^T}}{\left[B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right]} > 0, && (101c)\\
\lambda_{G^N} \equiv \frac{\partial \bar{\lambda}_i}{\partial G_i^N} &= -\frac{\left[B_{G^N} - \Phi_1 K_{G^N}\right]}{\left[B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right]} > 0. && (101d)
\end{aligned}$$

From (101), we obtain the following properties:

$$\begin{aligned}
\lambda_B \left[B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right] &= 1, && (102a)\\
\lambda_B B_{G^T} &= -\lambda_{G^T}, && (102b)\\
\lambda_B \left[B_{G^N} - \Phi_1 K_{G^N}\right] &= -\lambda_{G^N}. && (102c)
\end{aligned}$$

F.3 Formal Solutions for Temporary Fiscal Shocks

We assume that the small open economy is initially in steady-state equilibrium, denoted by the subscript $i = 0$:

$$\begin{aligned}
K_0 &= \tilde{K}_0 = K\left(\bar{\lambda}_0, G_0^N\right) = K\left(\lambda\left(K_0, B_0, G_0^T, G_0^N\right), G_0^N\right), && (103a)\\
B_0 &= \tilde{B}_0 = B\left(\bar{\lambda}_0, G_0^T, G_0^N\right) = B\left(\lambda\left(K_0, B_0, G_0^T, G_0^N\right), G_0^T, G_0^N\right), && (103b)\\
\lambda_0 &= \bar{\lambda}_0 = \lambda\left(K_0, B_0, G_0^T, G_0^N\right). && (103c)
\end{aligned}$$

We suppose now that government expenditure changes unexpectedly at time $t = 0$ from the original level G_0^T (resp. G_0^N) to level G_1^T (resp. G_1^N) over the period $0 \leq t < \mathcal{T}$, and reverts back at time $\mathcal{T}$ permanently to its initial level, $G_{\mathcal{T}}^T = G_2^T = G_0^T$ (resp. $G_{\mathcal{T}}^N = G_2^N = G_0^N$).

Period 1 ($0 \leq t < \mathcal{T}$)

Whereas the fiscal expansion is implemented, the economy follows unstable transitional paths:

$$\begin{aligned}
K(t) &= \tilde{K}_1 + B_1 e^{\nu_1 t} + B_2 e^{\nu_2 t}, && (104a)\\
P(t) &= \tilde{P}_1 + \omega_2^1 B_1 e^{\nu_1 t} + \omega_2^2 B_2 e^{\nu_2 t}, && (104b)\\
B(t) &= \tilde{B}_1 + \left[\left(B_0 - \tilde{B}_1\right) - \Phi_1 B_1 - \Phi_2 B_2\right] e^{r^\star t} + \\
&\quad + \Phi_1 B_1 e^{\nu_1 t} + \Phi_2 B_2 e^{\nu_2 t}, && (104c)
\end{aligned}$$

with the steady-state values $\tilde{K}_1$ and $\tilde{B}_1$ given by the following functions (set $i = 1$ into (94b)-(94c)):

$$\begin{aligned}\tilde{K}_1 &= K\left(\bar{\lambda}, G_1^N\right), &(105a)\\ \tilde{B}_1 &= B\left(\bar{\lambda}, G_1^T, G_1^N\right), &(105b)\end{aligned}$$

where the marginal utility of wealth remains constant over periods 1 and 2 at level $\bar{\lambda}_1 = \bar{\lambda}_2 = \bar{\lambda}$ after its initial jump at time $t = 0$.

Period 2 ($t \geq \mathcal{T}$)

Once government spending reverts back to its initial level, the economy follows stable paths

$$\begin{aligned}K(t) &= \tilde{K}_2 + B_1' e^{\nu_1 t}, &(106a)\\ P(t) &= \tilde{P}_2 + \omega_2^1 B_1' e^{\nu_1 t}, &(106b)\\ B(t) &= \tilde{B}_2 + \Phi_1 B_1' e^{\nu_1 t}, &(106c)\end{aligned}$$

with the steady-state values $\tilde{K}_2$ and $\tilde{B}_2$ given by the following functions (set $i = 2$ into (94b)-(94c)):

$$\begin{aligned}\tilde{K}_2 &= K\left(\bar{\lambda}, G_2^N\right), &(107a)\\ \tilde{B}_2 &= B\left(\bar{\lambda}, G_2^T, G_2^N\right). &(107b)\end{aligned}$$

During the transition period 1, the economy accumulates capital and foreign assets. Since this period is unstable, it would lead the nation to violate its intertemporal budget constraint. By contrast, the adjustment process taking place in period 2 is stable and must satisfy the economy's intertemporal budget constraint. At the same time, the zero-root problem requires the equilibrium value of marginal utility of wealth to adjust once-and-for-all when the shock hits the economy. So λ remains constant over the periods 1 and 2. The aim of the *two-step method* is to calculate the deviation of λ such that the country satisfies one single and overall intertemporal budget constraint, given the new relevant initial conditions, $K_{\mathcal{T}}$ and $B_{\mathcal{T}}$, prevailing when the shock ends and accumulated over the unstable period. Therefore, for the country to remain intertemporally solvent, we require:

$$B_{\mathcal{T}} - \tilde{B}_2 = \Phi_1\left(K_{\mathcal{T}} - \tilde{K}_2\right). \tag{108}$$

In order to determine the three constants B_1, B_2, and B_1', and the equilibrium value of marginal utility of wealth, we impose three conditions:

1. Initial conditions $K(0) = K_0$, $B(0) = B_0$ must be met.
2. Economic aggregates K and P remain continuous at time $\mathcal{T}$.
3. The intertemporal solvency constraint (108) must hold implying that the net foreign assets remain continuous at time $\mathcal{T}$.

Set $t = 0$ in solution (104a), and evaluating first at time $t = \mathcal{T}$, equate (104a) and (106a), (104b) and (106b):

$$\begin{aligned}\tilde{K}_1 + B_1 + B_2 &= K_0, &(109a)\\ \tilde{K}_1 + B_1 e^{\nu_1 \mathcal{T}} + B_2 e^{\nu_2 \mathcal{T}} &= \tilde{K}_2 + B_1' e^{\nu_1 \mathcal{T}}, &(109b)\\ \tilde{P}_1 + \omega_2^1 B_1 e^{\nu_1 \mathcal{T}} + \omega_2^2 B_2 e^{\nu_2 \mathcal{T}} &= \tilde{P}_2 + \omega_2^1 B_1' e^{\nu_1 \mathcal{T}}, &(109c)\end{aligned}$$

where we used the continuity condition.

Evaluating $K_{\mathcal{T}}$ and $B_{\mathcal{T}}$ from respectively (104a) and (104c), substituting into (108), and using functions of steady-state values $\tilde{K}_i$ and $\tilde{B}_i$ given by (103) (for $i = 0$), (105) (for $i = 1$), and (107) (for $i = 2$), the intertemporal solvency condition can be rewritten as

$$B\left(\bar{\lambda}, G_1^T, G_1^N\right) + \left[\left(B\left(\lambda_0, G_0^T, G_0^N\right) - B\left(\bar{\lambda}, G_1^T, G_1^N\right)\right) - \Phi_1 B_1 - \Phi_2 B_2\right] e^{r^\star \mathcal{T}} + \Phi_1 B_1 e^{\nu_1 \mathcal{T}}$$
$$+\Phi_2 B_2 e^{\nu_2 \mathcal{T}} - B\left(\bar{\lambda}, G_2^T, G_2^N\right) = \Phi_1 \left[K\left(\bar{\lambda}, G_1^N\right) + B_1 e^{\nu_1 \mathcal{T}} + B_2 e^{\nu_2 \mathcal{T}} - K\left(\bar{\lambda}, G_2^N\right)\right]. \quad (110)$$

Then, we approximate the steady-state changes with the differentials:

$$\tilde{K}_1 - \tilde{K}_0 \equiv K\left(\bar{\lambda}, G_1^N\right) - K\left(\lambda_0, G_0^N\right) = K_{\bar{\lambda}} \mathrm{d}\bar{\lambda} + K_{G^N} \mathrm{d}G^N, \quad (111a)$$
$$\tilde{K}_2 - \tilde{K}_1 \equiv K\left(\bar{\lambda}, G_2^N\right) - K\left(\bar{\lambda}, G_1^N\right) = -K_{G^N} \mathrm{d}G^N, \quad (111b)$$
$$\tilde{B}_1 - \tilde{B}_0 \equiv B\left(\bar{\lambda}, G_1^T, G_1^N\right) - B\left(\lambda_0, G_0^T, G_0^N\right) = B_{\bar{\lambda}} \mathrm{d}\bar{\lambda} + B_{G^T} \mathrm{d}G^T + B_{G^N} \mathrm{d}G^N, \quad (111c)$$
$$\tilde{B}_2 - \tilde{B}_1 \equiv B\left(\bar{\lambda}, G_2^T, G_2^N\right) - B\left(\bar{\lambda}, G_1^T, G_1^N\right) = -B_{G^T} \mathrm{d}G^T - B_{G^N} \mathrm{d}G^N, \quad (111d)$$

where $\mathrm{d}\bar{\lambda} \equiv \bar{\lambda} - \lambda_0$.

By substituting these expressions in (109) and (110), we obtain finally

$$B_1 + B_2 = -K_{\bar{\lambda}} \mathrm{d}\bar{\lambda} - K_{G^N} \mathrm{d}G^N, \quad (112a)$$
$$B_1 e^{\nu_1 \mathcal{T}} + B_2 e^{\nu_2 \mathcal{T}} - B_1' e^{\nu_1 \mathcal{T}} = -K_{G^N} \mathrm{d}G^N, \quad (112b)$$
$$\omega_2^1 B_1 e^{\nu_1 \mathcal{T}} + \omega_2^2 B_2 e^{\nu_2 \mathcal{T}} - \omega_2^1 B_1' e^{\nu_1 \mathcal{T}} = 0, \quad (112c)$$

and

$$B_1 \Upsilon_1 + B_2 \Upsilon_2 + B_{\bar{\lambda}} \mathrm{d}\bar{\lambda} = \Omega_1, \quad (113)$$

where we set

$$\Upsilon_1 \equiv \Phi_1, \quad (114a)$$
$$\Upsilon_2 \equiv \Phi_2 + \left(\Phi_1 - \Phi_2\right) e^{-\nu_1 \mathcal{T}}, \quad (114b)$$
$$\Omega_1 \equiv \left[\left(v_{g^j} - \Phi_1 K_{g^j}\right) e^{-r^\star \mathcal{T}} - v_{g^j}\right] \mathrm{d}g^j \quad j = T, N, \quad (114c)$$

where $K_{G^T} = 0$.

Case $k^N > k^T$

We write out some useful expressions

$$
\begin{aligned}
K_{\bar{\lambda}} &= -\frac{\tilde{C}^N}{\bar{\lambda}}\frac{\sigma_C}{\nu_2} < 0, && \text{(115a)}\\
K_{G^N} &= \frac{1}{\nu_2} > 0, && \text{(115b)}\\
B_{\bar{\lambda}} &= -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{\nu_2 r^\star}\left[(1-\alpha_C)\,\nu_2 - \alpha_C\nu_1\right] < 0, && \text{(115c)}\\
B_{G^N} &= -\frac{\tilde{P}\nu_1}{\nu_2 r^\star} > 0, && \text{(115d)}\\
(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}) &= -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{\nu_2 r^\star}\left[\nu_2 + \alpha_C\frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right] < 0, && \text{(115e)}\\
(B_{G^N} - \Phi_1 K_{G^N}) &= \frac{\tilde{P}}{\nu_2 r^\star}\left[\nu_2 + \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right] > 0, && \text{(115f)}\\
\Upsilon_2 &= -\tilde{P}\left[1 + \frac{\tilde{C}^N}{\tilde{P}}\frac{\sigma_C}{\nu_2}\omega_2^1 e^{-\nu_1\mathcal{T}}\right], && \text{(115g)}\\
B_{\bar{\lambda}} - \Upsilon_2 K_{\bar{\lambda}} &= -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{r^\star\nu_2}\left[\nu_2 + \alpha_C\frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1 e^{-\nu_1\mathcal{T}}\right] < 0, && \text{(115h)}\\
\Omega_1 K_{\bar{\lambda}} + B_{\bar{\lambda}} K_{G^N}\mathrm{d}G^N &= -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{r^\star\left(\nu_2\right)^2}\left\{\alpha_C\left[\nu_2 + \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]e^{-r^\star\mathcal{T}} + (1-\alpha_C)\,\nu_2\right\}\mathrm{d}G^N < 0, && \text{(115i)}
\end{aligned}
$$

and $B_{G^T} = 1/r^\star > 0$. We used the fact that $\tilde{k}^T\nu_2 + \tilde{k}^N\nu_1 = -\frac{W}{\tilde{P}}$ and the following expression:

$$
\Omega_1 = -\frac{1}{r^\star}\left(1 - e^{-r^\star\mathcal{T}}\right)\mathrm{d}G^T + \frac{\tilde{P}}{r^\star\nu_2}\left\{\nu_1 + \left[\nu_2 + \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]e^{-r^\star\mathcal{T}}\right\}\mathrm{d}G^N. \tag{116}
$$

Case $k^T > k^N$

We write out some useful expressions

$$\begin{aligned}
K_{\bar{\lambda}} &= -\frac{\tilde{C}^N}{\bar{\lambda}}\frac{\sigma_C}{\nu_1} > 0, && (117a)\\
K_{G^N} &= \frac{1}{\nu_1} < 0, && (117b)\\
B_{\bar{\lambda}} &= -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{\nu_1 r^\star}\left[(1-\alpha_C)\,\nu_1 - \alpha_C\nu_2\right] < 0, && (117c)\\
B_{G^N} &= -\frac{\tilde{P}\nu_2}{\nu_1 r^\star} > 0, && (117d)\\
\left(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right) &= -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{r^\star} < 0 && (117e)\\
\left(B_{G^N} - \Phi_1 K_{G^N}\right) &= \frac{\tilde{P}}{r^\star} > 0, && (117f)\\
\Upsilon_2 &= -\tilde{P}\left[1 + \frac{\tilde{C}^N}{\tilde{P}}\frac{\sigma_C}{\nu_1}\omega_2^2\left(1-e^{-\nu_1 \mathcal{T}}\right)\right] < 0, && (117g)\\
B_{\bar{\lambda}} - \Upsilon_2 K_{\bar{\lambda}} &= -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{r^\star\nu_1}\left[\nu_1 + \alpha_C\frac{r^\star}{\nu_1}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^2\left(1-e^{-\nu_1 \mathcal{T}}\right)\right] \gtrless 0, && (117h)\\
\Omega_1 K_{\bar{\lambda}} + B_{\bar{\lambda}} K_{G^N}\mathrm{d}G^N &= -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{r^\star\nu_1}\left[(1-\alpha_C) + \alpha_C e^{-r^\star \mathcal{T}}\right] > 0, && (117i)
\end{aligned}$$

and $B_{G^T} = 1/r^\star > 0$. We used the fact that $\tilde{k}^T\nu_1 + \tilde{k}^N\nu_2 = -\frac{W}{\tilde{P}}$ and the following expression:

$$\Omega_1 = -\frac{1}{r^\star}\left(1-e^{-r^\star \mathcal{T}}\right)\mathrm{d}G^T + \frac{\tilde{P}}{r^\star\nu_1}\left(\nu_2 + \nu_1 e^{-r^\star \mathcal{T}}\right)\mathrm{d}G^N. \qquad (118)$$

Case $k^N > k^T$

The solutions for a rise in the government expenditure on the traded good are given by:

$$\begin{aligned}
\frac{B_1}{\mathrm{d}G^T} &= \frac{\alpha_C\left(1-e^{-r^\star \mathcal{T}}\right)}{\tilde{P}\nu_2\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} > 0, && (119a)\\
\frac{B_2}{\mathrm{d}G^T} &= 0, && (119b)\\
\frac{B_1'}{\mathrm{d}G^T} &= \frac{B_1}{\mathrm{d}G^T}, && (119c)\\
\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} &= \lambda_{G^T}\left(1-e^{-r^\star \mathcal{T}}\right) > 0, && (119d)
\end{aligned}$$

where, from (112a), $\frac{B_1}{\mathrm{d}G^T}$ can be written also as follows

$$\frac{B_1}{\mathrm{d}G^T} = -K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp}. \qquad (120)$$

The solutions for a rise in the government expenditure on the non-traded good are given by:

$$\frac{B_1}{\mathrm{d}G^N} = -\frac{\left[\left(1-e^{-\nu_2 \mathcal{T}}\right) - \alpha_C\left(1-e^{-r^\star \mathcal{T}}\right)\right]}{\nu_2\left[1+\alpha_C \frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]}$$

$$= -\frac{(1-\alpha_C)\left(1-e^{-\nu_2 \mathcal{T}}\right) + \alpha_C\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)}{\nu_2\left[1+\alpha_C \frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} < 0, \quad (121a)$$

$$\frac{B_2}{\mathrm{d}G^N} = -\frac{e^{-\nu_2 \mathcal{T}}}{\nu_2} < 0, \quad (121b)$$

$$\frac{B_1'}{\mathrm{d}G^N} = \frac{B_1}{\mathrm{d}G^N} < 0, \quad (121c)$$

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \left(1-e^{-\nu_2 \mathcal{T}}\right)\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} + \frac{u_{cc}\tilde{P}}{(P_C)^2}\frac{\nu_2\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)}{\left[\nu_2+\alpha_C\frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]}$$

$$= \lambda_{G^N}\left\{\left(1-e^{-\nu_2 \mathcal{T}}\right) - \frac{\nu_2\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)}{\left[\nu_2+\frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]}\right\} \lesseqgtr 0, \quad (121d)$$

where we used expression (90b) to obtain (121d). From (112a), $\frac{B_1}{\mathrm{d}G^T}$ and $\frac{B_2}{\mathrm{d}G^T}$ can also be written as follows:

$$\frac{B_1}{\mathrm{d}G^N} + \frac{B_2}{\mathrm{d}G^N} = -K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - K_{G^N} \quad \text{and} \quad \frac{B_2}{\mathrm{d}G^N} = -K_{G^N}e^{-\nu_2 \mathcal{T}}. \quad (122)$$

Case $k^T > k^N$

The solutions for a rise in the government expenditure on the traded good are given by:

$$\frac{B_1}{\mathrm{d}G^T} = \frac{\alpha_C}{\nu_1\tilde{P}}\left(1-e^{-r^\star \mathcal{T}}\right) < 0, \quad (123a)$$

$$\frac{B_2}{\mathrm{d}G^T} = 0, \quad (123b)$$

$$\frac{B_1'}{\mathrm{d}G^T} = \frac{B_1}{\mathrm{d}G^T}, \quad (123c)$$

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = \lambda_{G^T}\left(1-e^{-r^\star \mathcal{T}}\right) > 0. \quad (123d)$$

The solutions for a rise in the government expenditure on the non-traded good are given by:

$$\frac{B_1}{\mathrm{d}G^N} = -\frac{1}{\nu_1}\left[(1-\alpha_C)+\alpha_C e^{-r^\star \mathcal{T}}\right],$$

$$= -\frac{1}{\nu_1}\left[(1-\alpha_C)\left(1-e^{-r^\star \mathcal{T}}\right)+e^{-r^\star \mathcal{T}}\right] > 0, \quad (124a)$$

$$\frac{B_2}{\mathrm{d}G^N} = 0, \quad (124b)$$

$$\frac{B_1'}{\mathrm{d}G^N} = \frac{B_1}{\mathrm{d}G^N} + K_{G^N}e^{-\nu_1 \mathcal{T}}$$

$$= -\frac{1}{\nu_1}\left[\left(1-e^{-\nu_1 \mathcal{T}}\right) - \alpha_C\left(1-e^{-r^\star \mathcal{T}}\right)\right] < 0, \quad (124c)$$

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \lambda_{G^N}\left(1-e^{-r^\star \mathcal{T}}\right) > 0. \quad (124d)$$

G Transitional Dynamics after a Rise in G^N

In this section, we investigate in details the dynamics of key variables after a permanent and temporary rise in G^N, considering both cases: $k^T > k^N$ and $k^N > k^T$. Transitional paths are depicted in Figures 2 and 4 for $k^T > k^N$ and $k^N > k^T$, respectively. To keep analytical tractability, we assume that labor supply is fixed, i.e. we set $\sigma_L = 0$. Since these two parameters do no affect qualitatively the results, we further assume that the non-traded sector is perfectly competitive, i.e. we set $\mu = 1$, and we set the rate of depreciation of physical capital to zero.

G.1 Long-Run Effects

We derive the ultimate steady-state changes of the economic key variables after a permanent rise in government spending on the non-traded good by differentiating the functions (94) w.r.t G^N:

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N}\right|_{perm} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} < 0, \tag{125a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} + K_{G^N} \gtrless 0 \quad \text{depending on whether} \quad k^N \gtrless k^T, \tag{125b}$$

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N}\right|_{perm} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} + B_{G^N} \lesseqgtr 0 \quad \text{depending on whether} \quad k^N \gtrless k^T, \tag{125c}$$

where analytical expressions are given by the set of equations (90) and (91).

We turn now to the long run changes of macroeconomic aggregates after a temporary fiscal expansion by considering two cases.

Case $k^N > k^T$

The equilibrium change of $\bar{\lambda}$ is:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \lambda_{G^N} \left\{ \left(1 - e^{-\nu_2 \mathcal{T}}\right) - \frac{\nu_2 \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)}{\left[\nu_2 + \frac{r^\star}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right]} \right\} < 0. \tag{126}$$

The sign of the change in the equilibrium value of the marginal utility of wealth can be determined by noticing that eq. (126) tends towards zero if we let $\mathcal{T}$ tend towards zero and tends towards λ_{G^N} if we let $\mathcal{T}$ tend towards ∞. In addition, the term in square brackets is an increasing and monotonic function of parameter $\mathcal{T}$. Therefore, the change in $\bar{\lambda}$ after a temporary rise in government spending lies in the range $[0, \lambda_{G^N}]$. Consequently, we can deduce that expression (126) has a positive sign.

Using the functions (94), we deduce the long run changes for the real consumption, the stock of physical capital, and the stock of traded bonds:

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N}\right|_{temp} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \tag{127a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \tag{127b}$$

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N}\right|_{temp} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \tag{127c}$$

where $C_{\bar{\lambda}} < 0$, $K_{\bar{\lambda}} < 0$, and $B_{\bar{\lambda}} < 0$.

The change of the period 1 steady-state value $\tilde{K}_1$ compared to its initial (given) value $\tilde{K}_0$ is given by:

$$
\begin{aligned}
\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^N}\right|_{temp} &= K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + K_{G^N}, \\
&= \frac{(1-\alpha_C) + \alpha_C \left[1 + \frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1 e^{\nu_1 \mathcal{T}}\right] e^{-r^\star \mathcal{T}}}{\nu_2 \left[1 + \alpha_C \frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right]} > 0, \qquad (128)
\end{aligned}
$$

where we have substituted expressions of $K_{\bar{\lambda}} < 0$ given by (115a), $\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\big|_{temp} > 0$ given by (126) and $K_{G^N} > 0$ given by (115b).

The change of the period 1 steady-state value $\tilde{B}_1$ compared to its initial (given) value $\tilde{B}_0$ is given by:

$$
\begin{aligned}
\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} &= B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + B_{G^N}, \qquad (129) \\
&= -\frac{\tilde{P}}{r^\star \nu_2} \frac{1}{\left[1 + \alpha_C \frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right]} \left\{ \left((1-\alpha_C)\nu_2 - \alpha_C \nu_1\right) \left[1 + \frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right] \left(1 - e^{-r^\star \mathcal{T}}\right) \right. \\
&\quad \left. + \left[1 + \alpha_C \frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right] \nu_1 \right\} \gtreqless 0,
\end{aligned}
$$

where we have substituted expressions of $B_{\bar{\lambda}} < 0$ given by (115c), $\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} > 0$ given by (126) and $B_{G^N} > 0$ given by (115d). We cannot sign eq. (129) because it is the result of two opposite effects. The first term on the RHS of (129) is negative and is an increasing function of parameter $\mathcal{T}$ and may be dominated by the second term B_{G^N} which is positive. We can infer that the shorter-lasting the rise in government expenditure, the more likely a higher steady-state value $\tilde{B}_1$ compared to its initial (given) value $\tilde{B}_0$.

It is interesting to compare the magnitudes of the long run changes in the stock of international assets between a permanent and a temporary fiscal expansion:

$$
\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N}\right|_{perm} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} + B_{G^N} \gtreqless B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N}\right|_{temp}, \qquad (130)
$$

where $B_{G^N} > 0$, $B_{\bar{\lambda}} < 0$ and $\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} = \lambda_{G^N} > 0$. The key factor that determines the magnitude of the long run change in the stock of foreign assets is the period of implementation of the government policy. More specifically, simulations indicate that there exists a time $\mathcal{T} = \acute{\mathcal{T}}$ for which the two changes are equal. For high durations of the policy, i. e. $\mathcal{T} > \acute{\mathcal{T}}$, the deterioration of the net foreign asset position features a greater magnitude after a temporary fiscal expansion compared to a permanent policy. This result is reversed when the public policy is implemented over a short period, say $\mathcal{T} < \acute{\mathcal{T}}$.

From steady-state changes following permanent and temporary rise in government expenditure on the non-traded good, we can deduce the following inequalities regardless of the length of the shock:

$$
\begin{aligned}
&\tilde{K}_{temp} < K_0 < \tilde{K}_{perm} < \tilde{K}_1, && (131a) \\
&\tilde{B}_{temp} < \tilde{B}_{perm} < B_0, \quad \text{if} \quad \mathcal{T} > \acute{\mathcal{T}}, && (131b) \\
&\tilde{B}_{perm} < \tilde{B}_{temp} < B_0, \quad \text{if} \quad \mathcal{T} < \acute{\mathcal{T}}. && (131c)
\end{aligned}
$$

Case $k^T > k^N$

The equilibrium change of $\bar{\lambda}$ is:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \lambda_{G^N}\left(1 - e^{-r^\star \mathcal{T}}\right) > 0. \tag{132}$$

From (132), we see that that the change of λ after a temporary change in G^N is smaller than that after a permanent increase in G^N but goes in the same direction. Hence we deduce the following inequality:

$$0 < \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm}. \tag{133}$$

From (94), we deduce steady-state changes of consumption, the stock of physical capital, and the stock of traded bonds:

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N}\right|_{temp} = C_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \tag{134a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{temp} = K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} > 0, \tag{134b}$$

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N}\right|_{temp} = B_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \tag{134c}$$

where $C_{\bar{\lambda}} < 0$, $K_{\bar{\lambda}} > 0$, and $B_{\bar{\lambda}} < 0$.

Changes of the period 1 steady-state values $\tilde{K}_1$ and $\tilde{B}_1$ compared to their initial (given) values K_0 and B_0 are given by :

$$\begin{aligned}
\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^N}\right|_{temp} &= K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + K_{G^N}, \\
&= \frac{(1-\alpha_C) + \alpha_C e^{-r^\star \mathcal{T}}}{\nu_1} < 0,
\end{aligned} \tag{135a}$$

$$\begin{aligned}
\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} &= B_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + B_{G^N}, \\
&= -\frac{\tilde{P}}{r^\star \nu_1}\left\{(1-\alpha_C)\, r^\star - \left[(1-\alpha_C)\,\nu_1 - \alpha_C \nu_2\right] e^{-r^\star \mathcal{T}}\right\} > 0,
\end{aligned} \tag{135b}$$

where we have evaluated the signs of (135a)-(135b) by making use of (117a)-(117d) and (91b).

From (133), because the change in the equilibrium value of $\bar{\lambda}$ following a temporary change in G^N is smaller than that after a permanent increase in G^N, by making use of (134b)-(134c), (125b)-(125c), and (135a)-(135b), we are able to deduce the following inequalities:

$$\tilde{K}_1 < \tilde{K}_{perm} < K_0 < \tilde{K}_{temp}, \tag{136a}$$

$$\tilde{B}_{temp} < B_0 < \tilde{B}_{perm} < \tilde{B}_1. \tag{136b}$$

G.2 Transitional Dynamics after a Permanent Increase in G^N

Case $k^N > k^T$

The initial jump of P is obtained by setting $t = 0$ in (104b) and by differentiating with respect to G^N:

$$\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{perm} = -\omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} > 0. \tag{137}$$

From the short run static solutions, and by substituting the change in the equilibrium value of the marginal utility of wealth and the initial jump of the relative price of the non-traded good, we get the initial jump of consumption:

$$\begin{aligned}
\left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{perm} &= C_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} + C_P\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{perm} = -\frac{\tilde{P}\left[\nu_2 + \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right] - \tilde{C}^N\sigma_C\omega_2^1}{P_C\left[\nu_2 + \alpha_C\frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} \\
&= \left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N}\right|_{perm} + \frac{(1-\alpha_C)}{P_C}\frac{\tilde{C}^N\sigma_C\omega_2^1}{\left[\nu_2 + \alpha_C\frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} < 0. \qquad (138)
\end{aligned}$$

From (138), we deduce the following inequality

$$\left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{perm} < \left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N}\right|_{perm} = C_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} < 0. \qquad (139)$$

The rise in the marginal utility of wealth and the initial appreciation in the relative price of the non-traded good lowers $C(0)$ below its steady-state value. Along the stable adjustment, real consumption rises:

$$\dot{C}(t) = -C\sigma_C\alpha_C\frac{\dot{P}(t)}{P(t)} > 0, \qquad (140)$$

where the relative price of the non-traded good depreciates along the stable adjustment when the non-traded sector is relatively more capital intensive. Otherwise, the relative price of the non-traded good's and thus the real consumption's temporal paths are flat.

The dynamics of the key economic variables after a permanent rise in government spending falling on the non-traded good are as follows:

$$\begin{aligned}
\dot{K}(t) &= -\nu_1\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} e^{\nu_1 t}\mathrm{d}G^N > 0, && (141a) \\
\dot{P}(t) &= -\nu_1\omega_2^1\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} e^{\nu_1 t}\mathrm{d}G^N < 0, && (141b) \\
\dot{B}(t) &= -\nu_1\Phi_1\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} e^{\nu_1 t}\mathrm{d}G^N < 0. && (141c)
\end{aligned}$$

Note that the long run changes of $\tilde{K}$ and $\tilde{B}$ are opposite to those after a permanent rise G^T.

Case $k^T > k^N$

If $k^T > k^N$, the initial change in the real consumption is solely affected by the change in the equilibrium value of the marginal utility of wealth and jumps immediately to its new lower steady-state level:

$$\left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{perm} = C_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} = \left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N}\right|_{perm} < 0. \qquad (142)$$

Over time, investment decreases and the stock of international assets rises:

$$\begin{aligned}
I(t) &= -\nu_1\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} e^{\nu_1 t}\mathrm{d}G^N < 0, && (143a) \\
CA(t) &= -\nu_1\Phi_1\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} e^{\nu_1 t}\mathrm{d}G^N > 0. && (143b)
\end{aligned}$$

As will be useful later, we calculate the slope of the trajectory after a permanent fiscal expansion in the (K, B)-space by differentiating the solutions for $B(t)$ and for $K(t)$ w.r.t time:

$$\frac{\mathrm{d}B(t)}{\mathrm{d}K(t)} = \frac{\nu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^N} e^{\nu_1 t}}{\nu_1 \frac{B_1}{\mathrm{d}G^N} e^{\nu_1 t}} = -\tilde{P} < 0. \tag{144}$$

where we used the fact that $\Phi_1 = -\tilde{P}$.

G.3 Transitional Dynamics after a Temporary Increase in G^N

Case $k^N > k^T$

First, we evaluate the constants $B_1/\mathrm{d}G^N$ and $B_2/\mathrm{d}G^N$:

$$\begin{aligned}
\frac{B_1}{\mathrm{d}G^N} &= -\frac{B_2}{\mathrm{d}G^N} - K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - K_{G^N}, \\
&= -\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} \left[\left(1 - e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1-\alpha_C}\right)\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)\right] < 0. \quad (145a) \\
\frac{B_2}{\mathrm{d}G^N} &= -K_{G^N} e^{-\nu_2 \mathcal{T}} = -\frac{e^{-\nu_2 \mathcal{T}}}{\nu_2} < 0. \quad (145b)
\end{aligned}$$

By evaluating the formal solution for $P(t)$ at time $t = 0$, differentiating with respect to G^N, and remembering that $\mathrm{d}\tilde{P}_1/\mathrm{d}G^N = 0$, we get the initial jump of P:

$$\begin{aligned}
\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{temp} &= \omega_2^1 \frac{B_1}{\mathrm{d}G^N} = -\omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} \left(1 - e^{-\nu_2 \mathcal{T}}\right) - \omega_2^1 \frac{\alpha_C \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)}{\left[\nu_2 + \frac{r^\star}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right]} \\
&= -\omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} \left[\left(1 - e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1-\alpha_C}\right)\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)\right] > 0, \quad (146)
\end{aligned}$$

where we have inserted the steady-state change of the capital stock after a permanent fiscal expansion falling on the non-traded good given by (90e). From (146), we can see that the magnitude of the initial appreciation in the real exchange after a temporary fiscal expansion may be magnified if the policy is implemented during a long period, i. e. for $\mathcal{T} > \frac{1}{\nu_1} \ln\left[\alpha_C\right]$.

By making use of the short run static solution (26) for C, we obtain the response of real consumption at time $t = 0$:

$$\left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{temp} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + C_P \left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{temp} < 0. \tag{147}$$

It is now convenient to evaluate the magnitude of the downward jump of real consumption after a temporary rise in G^N compared with that after a permanent fiscal expansion by computing the following expression:

$$\begin{aligned}
\left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{temp} - \left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{perm} &= C_{\bar{\lambda}} \left[\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm}\right] \\
&\quad + C_P \left[\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{temp} - \left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{perm}\right] \gtrless 0. \quad (148) \\
& \quad (149)
\end{aligned}$$

From (148), we deduce the following inequality:

$$\left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{perm} < \left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{temp} < 0. \tag{150}$$

The initial response of the investment flow following a temporary rise in G^N is given by:

$$\begin{aligned}
\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} &= \nu_1 \frac{B_1}{\mathrm{d}G^N} + \nu_2 \frac{B_2}{\mathrm{d}G^N} \\
&= -\nu_1 \left\{ \frac{(1-\alpha_C)\left(1-e^{-\nu_2 \mathcal{T}}\right) + \alpha_C \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)}{\left[\nu_2 + \alpha_C \frac{r^\star}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right]} \right\} - e^{-\nu_2 \mathcal{T}}, \\
&= -\nu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} \left[\left(1-e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1-\alpha_C}\right)\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)\right] - e^{-\nu_2 \mathcal{T}} \gtrless 0.
\end{aligned} \tag{151}$$

The sign of expression (151) is not clear-cut. As investment plays the role of clearing the non-traded goods market, its sign depends on the jumps of the relative price of the non-traded good and of the marginal utility of wealth. On the one hand, the relative price of the non-traded good appreciates which raises the return on domestic capital by reducing k^N. On the other hand, the increase in P raises the capital user cost. The latter effect is larger, the shorter-living the fiscal shock.

To derive a more easily interpretable expression for the initial reaction of investment after a temporary rise in G^N, we first linearize the non-traded good market clearing condition in the neighborhood of the steady-state:

$$I(t) - \tilde{I} = Y_K^N \left(K(t) - \tilde{K}\right) + \left(Y_P^N - C_P^N\right)\left(P(t) - \tilde{P}\right).$$

Using the fact that $\mathrm{d}\tilde{I} = Y_K^N \mathrm{d}\tilde{K} + \left(Y_P^N - C_P^N\right)\mathrm{d}\tilde{P} - C_{\bar{\lambda}}^N \mathrm{d}\bar{\lambda}|_{temp} - \mathrm{d}G^N$, and evaluating the expression above at time $t = 0$, we get:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} = \left(Y_P^N - C_P^N\right) \left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{temp} + \sigma_C \frac{\tilde{C}^N}{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - 1. \tag{152}$$

Using the fact that $\mathrm{d}\tilde{P} = 0$, we evaluate the initial jump of P which is given by:

$$\begin{aligned}
\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{temp} &= \omega_2^1 \frac{\mathrm{d}B_1}{\mathrm{d}G^N} = -\omega_2^1 \left[K_{G^N}\left(1-e^{-\nu_2 \mathcal{T}}\right) + K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp}\right], \\
&= \omega_2^1 \left[-\frac{\left(1-e^{-\nu_2 \mathcal{T}}\right)}{\nu_2} + \frac{\sigma_C}{\nu_2} \frac{\tilde{C}^N}{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp}\right],
\end{aligned} \tag{153}$$

where we substituted $K_{G^N} = 1/\nu_2$ and $K_{\bar{\lambda}} = -\frac{\sigma_C}{\nu_2}\frac{\tilde{C}^N}{\bar{\lambda}}$. Substituting (153) into (152) and using the fact that $\omega_2^1 = \frac{\nu_1 - \nu_2}{\left(Y_P^N - C_P^N\right)}$, the initial reaction of investment finally rewrites as:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} = \left(\frac{\nu_2 - \nu_1}{\nu_2}\right)\left(1-e^{-\nu_2 \mathcal{T}}\right) + \frac{\sigma_C \tilde{C}^N}{\bar{\lambda}} \frac{\nu_1}{\nu_2} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - 1. \tag{154}$$

By differentiating the formal solution (104c) over period 1 for $B(t)$ with respect to time, then evaluating the resulting expressions at $t = 0$, and differentiating with respect to G^N, we obtain the initial response of the current account following a temporary fiscal expansion:

$$\begin{aligned}
\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} &= r^\star \left\{-\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \Phi_1 \frac{B_1}{\mathrm{d}G^N} - \Phi_2 \frac{B_2}{\mathrm{d}G^N}\right\} \\
&\quad + \nu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^N} + \nu_2 \Phi_2 \frac{B_2}{\mathrm{d}G^N}.
\end{aligned} \tag{155}$$

In order to simplify the solution (155), we rewrite the term in square brackets as follows

$$
\begin{aligned}
& -\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \left[\Phi_1 \frac{B_1}{\mathrm{d}G^N} + \Phi_2 \frac{B_2}{\mathrm{d}G^N}\right] \\
= & -\left[B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right] \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - \left[B_{G^N} - \Phi_1 K_{G^N}\right] + \left[\Phi_1 - \Phi_2\right] \frac{B_2}{\mathrm{d}G^N}, \\
= & -\frac{\lambda_{G^N}}{\lambda_B} \left\{ \left(1 - e^{-\nu_2 \mathcal{T}}\right) - \frac{\nu_2 \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)}{\left[\nu_2 + \frac{r^\star}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right]} \right\} + \frac{\lambda_{G^N}}{\lambda_B} + \frac{1}{\nu_2} \tilde{C}^N \sigma_C \omega_2^1 K_{G^N} e^{-\nu_2 \mathcal{T}}, \\
= & \frac{\lambda_{G^N}}{\lambda_B} e^{\nu_2 \mathcal{T}} - \frac{\tilde{P}}{r^\star} e^{-r^\star \mathcal{T}} + \frac{\tilde{P}}{\nu_2 r^\star} \left[\nu_2 + \frac{r^\star}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right] e^{-\nu_2 \mathcal{T}}, \\
= & -\frac{\tilde{P}}{r^\star} e^{-r^\star \mathcal{T}} < 0, \qquad (156)
\end{aligned}
$$

where we have substituted the expression of the change in the equilibrium value of the marginal utility of wealth given by (121d), we made use of properties (102), expression (115f) and inserted these useful expressions:

$$
\begin{aligned}
\frac{B_1}{\mathrm{d}G^N} &= -\frac{B_2}{\mathrm{d}G^N} - K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - K_{G^N} < 0, \\
\Phi_1 - \Phi_2 &= -\frac{1}{\nu_2} \tilde{C}^N \sigma_C \omega_2^1 > 0, \\
\frac{B_2}{\mathrm{d}G^N} &= -K_{G^N} e^{-\nu_2 \mathcal{T}} < 0, \\
\frac{\left(B_{G^N} - \Phi_1 K_{G^N}\right)}{\left[\nu_2 + \frac{r^\star}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right]} &= \frac{\tilde{P}}{\nu_2 r^\star} > 0.
\end{aligned}
$$

By inserting (156) into (155), the expression of the initial response of the current account reduces to:

$$
\begin{aligned}
\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} = & \ \nu_1 \tilde{P} \left(1 + \frac{1}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right) \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} \left[\left(1 - e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1 - \alpha_C}\right) \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)\right] \\
& -\tilde{P} e^{-r^\star \mathcal{T}} + \tilde{P} e^{-\nu_2 \mathcal{T}}, \\
= & -\tilde{P} \left(1 + \frac{1}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right) \left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{perm} \left[\left(1 - e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1 - \alpha_C}\right) \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)\right] \\
& -\tilde{P} \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right) < 0, \qquad (157)
\end{aligned}
$$

where we simplified several expressions as follows:

$$
\begin{aligned}
K_{\bar{\lambda}} \frac{u_{CC} \tilde{P}}{P_C^2} \nu_2 &= \frac{\tilde{P} \tilde{C}^N}{P_C \tilde{C}} = \alpha_C > 0, \\
\nu_2 \Phi_2 - \nu_1 \Phi_1 &= -\tilde{P} \nu_2 + \tilde{P} \nu_1 \left(1 + \frac{1}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right) < 0.
\end{aligned}
$$

To derive a more easily interpretable expression for the initial reaction of the current account after a temporary rise in G^N, we use eq. (145a):

$$
\begin{aligned}
\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} &= -\tilde{P}\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right) + \nu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^N}, \\
&= -\tilde{P}\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right) - \nu_1 \Phi_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} \left[\left(1 - e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1-\alpha_C}\right)\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)\right], \\
&= -\tilde{P}\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right) \\
&\quad -\nu_1 \Phi_1 \frac{(1-\alpha_C)}{\nu_2\left(1-\alpha_C \tilde{\Psi}\right)} \left[\left(1 - e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1-\alpha_C}\right)\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)\right] < 0,
\end{aligned} \tag{158}
$$

where $0 < \tilde{\Psi} \equiv -\frac{r^\star}{\nu_2^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1 < 1$.

Now, we investigate the dynamics for $K(t)$ and $P(t)$ over the unstable period $(0, \mathcal{T})$, say period 1:

$$
\begin{aligned}
\dot{K}(t) &= \nu_1 \frac{B_1}{\mathrm{d}G^N} e^{\nu_1 t} \mathrm{d}G^N + \nu_2 \frac{B_2}{\mathrm{d}G^N} e^{\nu_2 t} \mathrm{d}G^N \gtreqless 0, && (159a) \\
\dot{P}(t) &= \nu_1 \omega_2^1 \frac{B_1}{\mathrm{d}G^N} e^{\nu_1 t} \mathrm{d}G^N < 0, && (159b)
\end{aligned}
$$

where $B_1/\mathrm{d}G^N < 0$, $B_2/\mathrm{d}G^N < 0$, and $\omega_2^1 < 0$. As it can be seen from (159a), investment dynamics are the result of two opposite forces. If the initial investment flow is positive, it must be negative at time $\tilde{t}$ along the trajectory:

$$
\tilde{t} = \frac{1}{\nu_1 - \nu_2} \ln\left[-\frac{\nu_2 B_2/\mathrm{d}G^N}{\nu_1 B_1/\mathrm{d}G^N}\right], \tag{160}
$$

where the term in square brackets is less than one under the condition that the initial investment flow is positive (see eq. (151)), otherwise the trajectory for investment is monotonic.

The current account dynamics over period 1 are described by the following equation:

$$
CA(t) = \left[\tilde{P} e^{-\nu_2(\mathcal{T}-t)}\left(1 - e^{-\nu_1(\mathcal{T}-t)}\right) + \nu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^N} e^{\nu_1 t}\right] \mathrm{d}G^N < 0. \tag{161}
$$

We turn now to the analysis of transitional dynamics over the stable period 2. By making use of standard methods, the adjustments of the stock of physical capital, the relative price of non tradables P and the stock of international assets are driven by the following equations:

$$
\begin{aligned}
\dot{K}(t) &= \nu_1 \frac{B_1'}{\mathrm{d}G^N} \mathrm{d}G^N e^{\nu_1 t} > 0, && (162a) \\
\dot{P}(t) &= \nu_1 \omega_2^1 \frac{B_1'}{\mathrm{d}G^N} \mathrm{d}G^N e^{\nu_1 t} < 0, && (162b) \\
\dot{B}(t) &= \nu_1 \Phi_1 \frac{B_1'}{\mathrm{d}G^N} \mathrm{d}G^N e^{\nu_1 t} < 0. && (162c)
\end{aligned}
$$

Evaluate (162c) at time t^+, and calculate $\mathrm{d}CA(\mathcal{T}) = CA(\mathcal{T}^+) - CA(\mathcal{T}^-)$, we can see that the current account is continuous in the neighborhood of time $\mathcal{T}$. Thus we have $CA(\mathcal{T}^-) = CA(\mathcal{T}^+)$. Performing the same procedure of investment, we obtain:

$$
\frac{\mathrm{d}I(\mathcal{T})}{\mathrm{d}G^N} = -\nu_2 \frac{B_2}{\mathrm{d}G^N} e^{\nu_2 \mathcal{T}} = 1. \tag{163}
$$

When the policy is removed at time $\mathcal{T}$, i. e. government spending falls by an amount equals to $\mathrm{d}G^N(\mathcal{T}) \equiv G_2^N - G_1^N \equiv -\mathrm{d}G^N$, investment must rise to guarantee that the market-clearing condition holds at time $\mathcal{T}$.

Case $k^T > k^N$

Like after a permanent fiscal expansion, an unexpected transitory rise in government spending on the non-traded good leaves unaffected the relative price of the non-traded good both in the short run and in the long run. To evaluate the investment dynamics, we differentiate the solution for $K(t)$ given by (104a) with respect to time, evaluate the resulting expression at time $t = 0$, and then differentiate with respect to G^N, keeping in mind that $B_2/\mathrm{d}G^N = 0$ if $k^T > k^N$:

$$\begin{aligned}
\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} &= \nu_1 \frac{B_1}{\mathrm{d}G^N} = -\nu_1 \frac{1}{\nu_1}\left[(1-\alpha_C)\left(1-e^{-r^\star \mathcal{T}}\right) + e^{-r^\star \mathcal{T}}\right], \\
&= \alpha_C\left(1-e^{-r^\star T}\right) - 1 < 0, \\
&= \left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{perm}\left(1-e^{-r^\star \mathcal{T}}\right) - e^{-r^\star \mathcal{T}} < 0.
\end{aligned} \tag{164}$$

Applying standard methods, the initial response of the current account following a temporary fiscal expansion on the non-traded good is given by:

$$\begin{aligned}
\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} &= r^\star\left\{-\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \Phi_1\frac{B_1}{\mathrm{d}G^N}\right\} + \nu_1\Phi_1\frac{B_1}{\mathrm{d}G^N}, \\
&= \tilde{P}\left(1-\alpha_C\right)\left(1-e^{-r^\star \mathcal{T}}\right) > 0,
\end{aligned} \tag{165}$$

where $\nu_1\Phi_1\frac{B_1}{\mathrm{d}G^N} = \tilde{P}\left[(1-\alpha_C)\left(1-e^{-r^\star \mathcal{T}}\right) + e^{-r^\star \mathcal{T}}\right]$.

In deriving (165), we have also simplified the term in square braces as follows:

$$\begin{aligned}
&-\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \Phi_1\frac{B_1}{\mathrm{d}G^N} \\
&= -\left\{\left[\left(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right)\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + \left(B_{G^N} - \Phi_1 K_{G^N}\right)\right]\right\}, \\
&= \frac{\lambda_{G^N}}{\lambda_B}e^{-r^\star \mathcal{T}} = -\frac{\tilde{P}}{r^\star}e^{-r^\star \mathcal{T}} < 0.
\end{aligned} \tag{166}$$

We investigate the dynamics of the stocks of physical capital and traded bonds by taking the time derivative of formal solutions prevailing over period 1:

$$\begin{aligned}
I(t) &= \dot{K}(t) = \nu_1\frac{B_1}{\mathrm{d}G^N}\mathrm{d}G^N e^{\nu_1 t}, \\
&= -\nu_1\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm}\left(1-e^{-r^\star \mathcal{T}}\right)\mathrm{d}G^N e^{\nu_1 t} - e^{-r^\star \mathcal{T}}\mathrm{d}G^N e^{\nu_1 t} < 0,
\end{aligned} \tag{167}$$

and

$$\begin{aligned}
CA(t) &= -r^\star\left[\left(n\left(\bar{\lambda}, G_1^N\right) - B\left(\lambda_0, G_0^N\right)\right) + \Phi_1\frac{B_1}{\mathrm{d}G^N}\right]\mathrm{d}G^N e^{r^\star t} + \nu_1\Phi_1\frac{B_1}{\mathrm{d}G^N}\mathrm{d}G^N e^{\nu_1 t}, \\
&= \tilde{P}\left[(1-\alpha_C)\left(1-e^{-r^\star \mathcal{T}}\right)e^{\nu_1 t} - e^{-r^\star \mathcal{T}}\left(e^{r^\star t} - e^{\nu_1 t}\right)\right]\mathrm{d}G^N \gtrless 0.
\end{aligned} \tag{168}$$

There exists a time $t = \acute{t}$ such that the current account changes of sign:

$$\acute{t} = -\frac{1}{\nu_2}\ln\left[\frac{e^{-r^\star \mathcal{T}}}{(1-\alpha_C)\left(1-e^{-r^\star \mathcal{T}}\right) + e^{-r^\star \mathcal{T}}}\right], \tag{169}$$

where the term in square brackets is positive and lower than one. Over period 1, the current account improves first while the negative investment flow more than outweighs the *smoothing* effect. At time $\acute{t}$, these two effects cancel each other and after this date, the current account deteriorates as the smoothing behavior predominates, such that $CA\left(\mathcal{T}^{-}\right) < 0$. To see it more formally, we evaluate (168) at time $\mathcal{T}^{-}$:

$$CA\left(T^{-}\right) = \tilde{P}e^{\nu_1 \mathcal{T}}\left[\left(1 - e^{-\nu_1 \mathcal{T}}\right) - \alpha_C\left(1 - e^{-r^{\star}\mathcal{T}}\right)\right]\mathrm{d}G^N < 0. \tag{170}$$

At time $\mathcal{T}^{-}$, the investment flow is also negative:

$$I\left(\mathcal{T}^{-}\right) = -e^{-\nu_2 \mathcal{T}}\left[1 - (1-\alpha_C)\left(1 - e^{r^{\star}\mathcal{T}}\right)\right] < 0. \tag{171}$$

We have now to compare the slope of the trajectory after a transitory fiscal expansion over period $0 \leq t < \acute{t}$ in the (K, B)-space with the slope of the trajectory after a permanent fiscal expansion:

$$\begin{aligned}
\frac{\mathrm{d}B(t)}{\mathrm{d}K(t)} &= \frac{-\tilde{P}e^{-r^{\star}(\mathcal{T}-t)} + \nu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^N}e^{\nu_1 t}}{\nu_1 \frac{B_1}{\mathrm{d}G^N}e^{\nu_1 t}}, \\
&= -\frac{\tilde{P}\left\{\left[(1-\alpha_C)\left(1-e^{-r^{\star}\mathcal{T}}\right) + e^{-r^{\star}\mathcal{T}}\right]e^{\nu_1 t} - e^{-r^{\star}(\mathcal{T}-t)}\right\}}{\left[(1-\alpha_C)\left(1-e^{-r^{\star}\mathcal{T}}\right) + e^{-r^{\star}\mathcal{T}}\right]e^{\nu_1 t}},
\end{aligned} \tag{172}$$

where we have substituted the expression of the constant $B_1/\mathrm{d}G^N$. Over period $0 \leq \acute{t} < t$, the numerator is positive and the denominator is negative. Thus the slope of the trajectory is negative in the (K, B)-space. Comparing the terms in numerator and in denominator of (172), it is straightforward to show that the slope in absolute terms is lower than $\tilde{P}$. Therefore, the slope is negative and lower (in absolute terms) than the slope of the trajectory after a permanent fiscal expansion (equal to $-\tilde{P}$).

We turn now to the investigation of transitional dynamics of key macroeconomic variables over the stable period, say period 2. By adopting the standard procedure, we get:

$$I(t) = \dot{K}(t) = \nu_1 \frac{B_1'}{\mathrm{d}G^N}\mathrm{d}G^N e^{\nu_1 t} > 0 \tag{173a}$$

$$CA(t) = \dot{B}(t) = \nu_1 \Phi_1 \frac{B_1'}{\mathrm{d}G^N}\mathrm{d}G^N e^{\nu_1 t} < 0. \tag{173b}$$

Since the period 2 is a stable period, the dynamics are monotonic. If we can determine the sign of (173) at time $t = \mathcal{T}^{+}$, we are able to evaluate the transitional dynamics over the entire period:

$$I\left(\mathcal{T}^{+}\right) = -\left[(1-\alpha_C)\left(e^{\nu_1 \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right) - \left(1 - e^{-\nu_2 \mathcal{T}}\right)\right]\mathrm{d}G^N > 0, \tag{174a}$$

$$CA\left(\mathcal{T}^{+}\right) = \tilde{P}\left[(1-\alpha_C)\left(e^{\nu_1 \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right) - \left(1 - e^{-\nu_2 \mathcal{T}}\right)\right]\mathrm{d}G^N < 0. \tag{174b}$$

From (170) and (174b), we deduce that the current account is continuous in the neighborhood of $\mathcal{T}$, such that $CA\left(\mathcal{T}^{-}\right) = CA\left(\mathcal{T}^{+}\right) < 0$. At the opposite, from (171) and (174a), we see that investment is not continuous in the neighborhood of T since at this date, it must clear the non tradable market. To see it formally, we write the non tradable clearing market condition at time $\mathcal{T}^{-}$ and at time $\mathcal{T}^{+}$:

$$I\left(T^{-}\right) = Y^N\left[K\left(\mathcal{T}^{-}\right), P\left(\mathcal{T}^{-}\right)\right] - C^N\left[\lambda\left(\mathcal{T}^{-}\right), P\left(\mathcal{T}^{-}\right)\right] - G_1^N < 0, \tag{175a}$$

$$I\left(\mathcal{T}^{+}\right) = Y^N\left[K\left(\mathcal{T}^{+}\right), P\left(\mathcal{T}^{+}\right)\right] - C^N\left[\lambda\left(\mathcal{T}^{+}\right), P\left(\mathcal{T}^{+}\right)\right] - G_2^N > 0, \tag{175b}$$

where $G_2^N = G_0^N$. Goods market equilibrium is subject to two discrete perturbations: one at time $t = 0$ when the government raises the public spending, the other at time $t = \mathcal{T}$ when

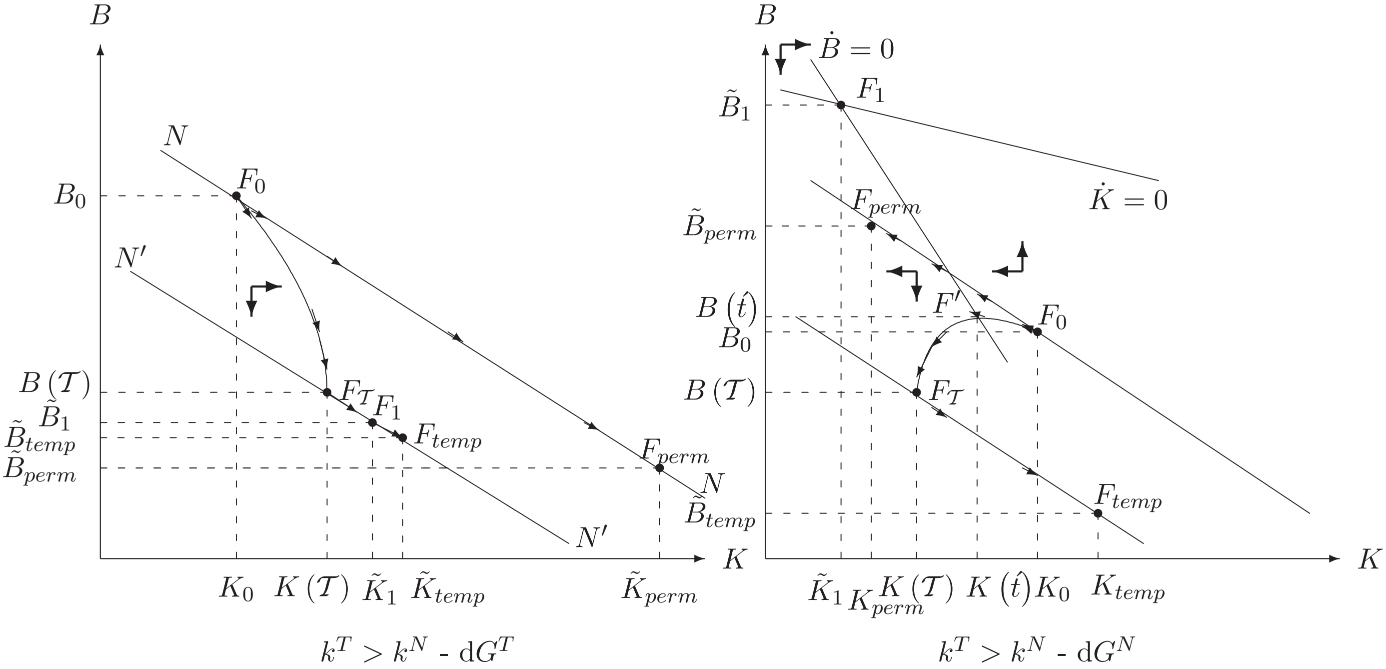


Figure 2: Permanent Vs. Temporary Increase in G^j - $k^T > k^N$

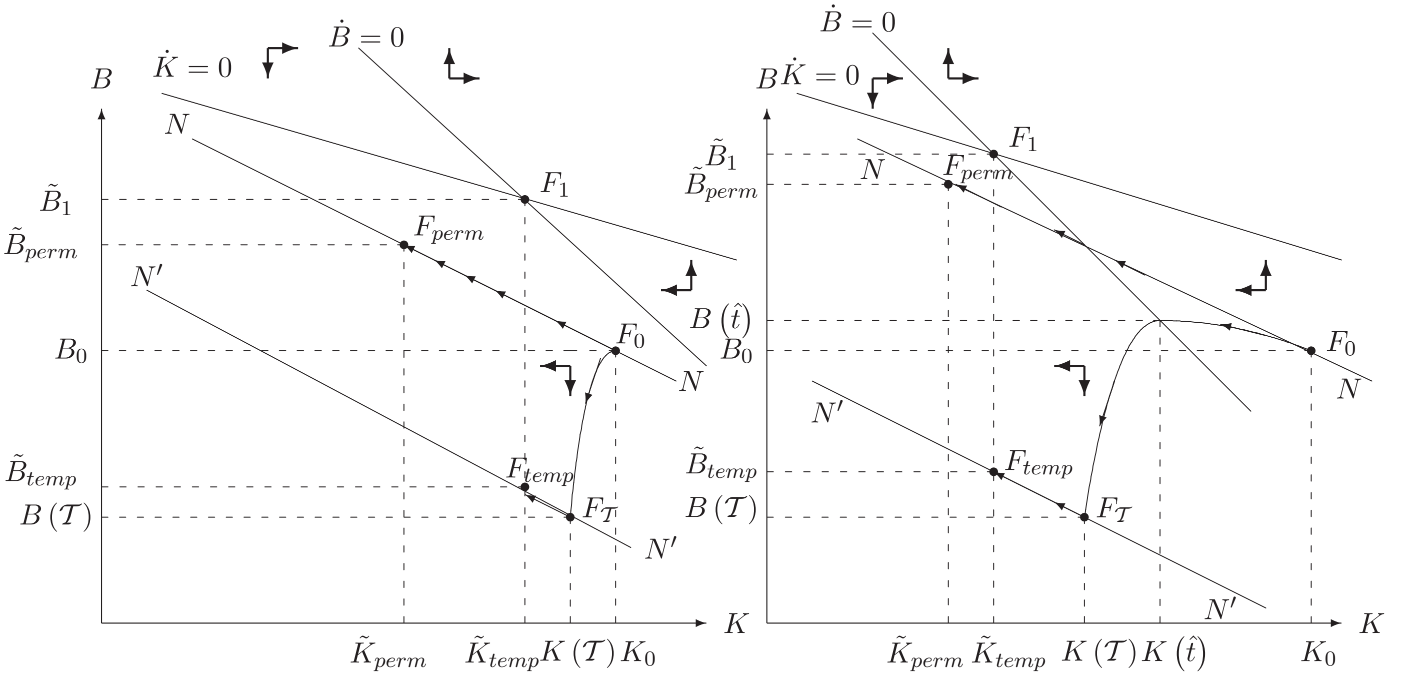
the policy is permanently removed. Since capital is a predetermined variable, it cannot jump neither at time $t = 0$ or at time $t = \mathcal{T}$. In addition, the marginal utility of wealth jumps at time $t = 0$ and remains constant from thereon. So we get $\bar{\lambda} = \lambda\left(\mathcal{T}^-\right) = \lambda\left(\mathcal{T}^+\right)$. Finally, when the tradable good sector is relatively more capital intensive, a rise in government spending leaves unaffected the relative price of the non-traded good both in the sort-run and in the long run, such that $\tilde{P} = P\left(\mathcal{T}^-\right) = P\left(\mathcal{T}^+\right)$. With output constrained at time $\mathcal{T}$ by the capital stock and by the relative price of the non-traded good, it therefore follows from (175) that for the market-clearing condition to hold, we must have

$$\mathrm{d}I\left(\mathcal{T}\right) = \mathrm{d}\dot{K}\left(\mathcal{T}\right) = -\mathrm{d}G^N\left(\mathcal{T}\right) = \mathrm{d}G^N > 0, \tag{176}$$

where $\mathrm{d}G^N\left(\mathcal{T}\right) \equiv G_2^N - G_1^N \equiv G_0^N - G_1^N \equiv -\mathrm{d}G^N$. Thus, the non-traded goods market equilibrium is maintained though the investment in physical capital, $\dot{K}\left(\mathcal{T}\right)$. Since at time $\mathcal{T}$, government expenditure reverts back to its original level, the investment flow changes of sign and turns out to be positive as a greater share of the non tradable production $\left(Y^N\right)$ may be allocated to investment (I) since the global consumption $\left(C^N + G^N\right)$ falls.

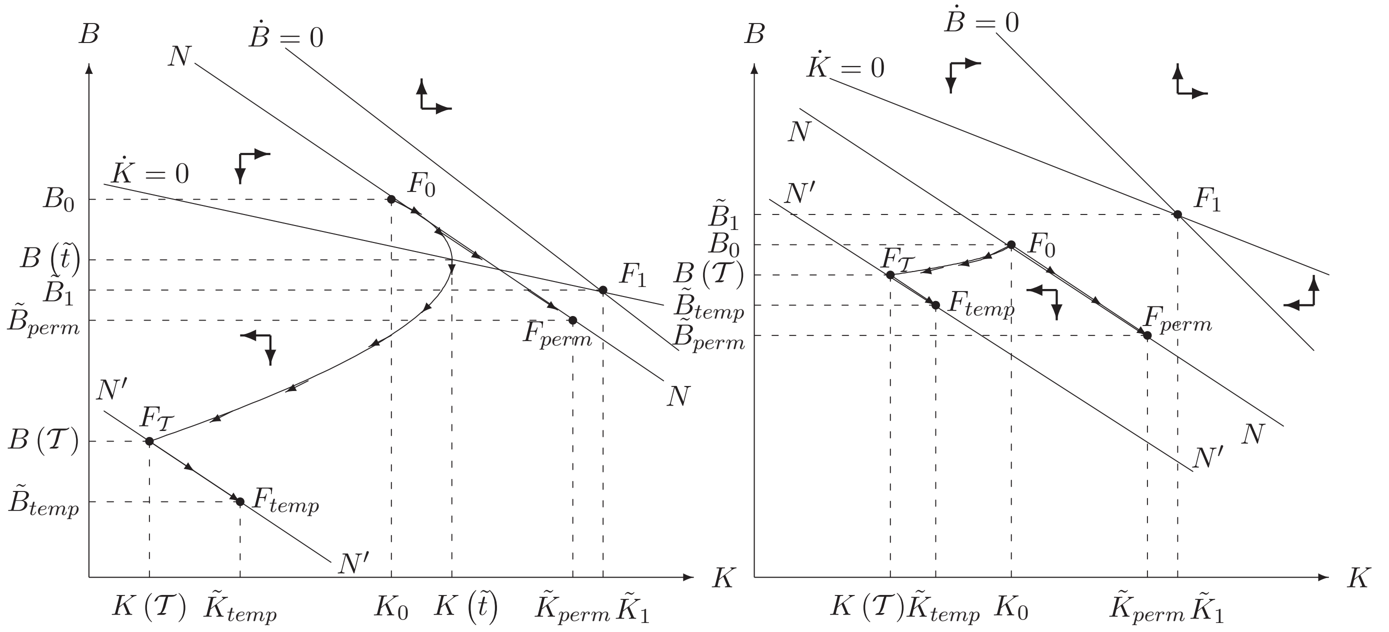
H Transitional Dynamics after a Rise in G^T

In the text, we consider only an increase in G^N. In this section, we analyze the effects of an increase in G^T. Hence, we provide details on the dynamics of key variables after a permanent and temporary rise in G^T, considering both cases: $k^T > k^N$ and $k^N > k^T$. Transitional paths are depicted in Figures 2 and 3 for $k^T > k^N$ and $k^N > k^T$, respectively. To keep analytical tractability, we assume that labor supply is fixed, i.e. we set $\sigma_L = 0$. Since these two parameters do no affect qualitatively the results, we further assume that the non-traded sector is perfectly competitive, i.e. we set $\mu = 1$, and we set the rate of depreciation of physical capital to zero.



$k^N > k^T$ - dG^T - $\mathcal{T} < \hat{\mathcal{T}}$ $\qquad$ $k^N > k^T$ - dG^T - $\mathcal{T} > \hat{\mathcal{T}}$

Figure 3: Permanent Vs. Temporary increase in G^T - $k^N > k^T$



$k^N > k^T$ - dG^N - $\mathcal{T} > \acute{\mathcal{T}}$ $\qquad$ $k^N > k^T$ - dG^N - $\mathcal{T} < \acute{\mathcal{T}}$

Figure 4: Permanent Vs. Temporary Increase in G^N - $k^N > k^T$

H.1 Long-Run Effects

It is convenient to determine first the long run changes of the real consumption, the stock of physical capital and the stock of foreign assets following a permanent rise in government spending on the traded good by differentiating (26) and (94):

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T}\right|_{perm} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < 0, \tag{177a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} \lesseqgtr 0 \quad \text{depending on whether} \quad k^N \gtreqless k^T, \tag{177b}$$

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} + B_{G^T} \gtreqless 0 \quad \text{depending on whether} \quad k^N \gtreqless k^T, \tag{177c}$$

where $C_{G^T} = 0$ and $K_{G^T} = 0$. Expressions of the steady-state changes are given by the set of equations (88) and (89).

We compare the once-for-all jump of the marginal utility of wealth after a permanent increase in public spending on the traded good with respect to its change after a permanent rise:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) = \lambda_{G^T} \left(1 - e^{-r^\star \mathcal{T}}\right) > 0. \tag{178}$$

We now evaluate the long run changes of key economic variables after a temporary fiscal shock by differentiating (26) and (94). Since the signs of expressions depend crucially on the sectoral capital intensities, we consider two cases.

Case $k^N > k^T$

When the non-traded sector is relatively more capital intensive, the variations of macroeconomic aggregates in the long run are given by:

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T}\right|_{temp} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = C_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < 0, \tag{179a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = K_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < 0, \tag{179b}$$

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{temp} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = B_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < 0, \tag{179c}$$

where $C_{\bar{\lambda}} < 0$, $K_{\bar{\lambda}} < 0$ (if $k^N > k^T$), and $B_{\bar{\lambda}} < 0$.

The changes of the period 1 steady-state values $\tilde{K}_1$ and $\tilde{B}_1$ compared to their initial (given) values K_0 and B_0 are given by :

$$\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} < 0, \tag{180a}$$

$$\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} + B_{G^T} > 0, \tag{180b}$$

where $K_{\bar{\lambda}} < 0$, $B_{\bar{\lambda}} < 0$ and $B_{G^T} > 0$. From (177b)-(177c), (179b)-(179c), and (180a)-(180b), we are able to deduce the following inequalities:

$$\tilde{K}_{perm} < \tilde{K}_1 = \tilde{K}_{temp} < K_0, \tag{181a}$$

$$\tilde{B}_{temp} < B_0 < \tilde{B}_{perm} < \tilde{B}_1. \tag{181b}$$

Case $k^T > k^N$

When the traded sector is relatively more capital intensive, the variations of macroeconomic aggregates in the long run are given by

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T}\right|_{temp} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = C_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < 0, \tag{182a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = K_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} > 0, \tag{182b}$$

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{temp} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = B_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < 0. \tag{182c}$$

It is interesting to compare the magnitudes of the long run changes in the stock of international assets between a permanent and a temporary fiscal expansion:

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} = B_{\bar{\lambda}}\lambda_{G^T} + B_{G^T} \gtreqless B_{\bar{\lambda}}\lambda_{G^T} \left(1 - e^{-r^\star \mathcal{T}}\right) = \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{temp}. \tag{183}$$

The key factor that determines the magnitude of the long run change in the stock of foreign assets is the period of implementation of the government policy. More specifically, there exists a time $\mathcal{T} = \tilde{\mathcal{T}}$ for which the two changes are equal which is given by

$$\tilde{\mathcal{T}} = \frac{1}{r^\star} \ln \left[-\frac{B_{\bar{\lambda}}\lambda_{G^T}}{B_{G^T}} \right]. \tag{184}$$

As the fiscal shock is more persistent, i. e. $\mathcal{T} > \tilde{\mathcal{T}}$, the external asset position deteriorates more than after a permanent fiscal shock. We can summarize our results as follows:

$$\tilde{B}_{temp} < \tilde{B}_{perm} < B_0 \quad \text{if} \quad \mathcal{T} > \tilde{\mathcal{T}}, \tag{185a}$$

$$\tilde{B}_{perm} < \tilde{B}_{temp} < B_0 \quad \text{if} \quad \mathcal{T} < \tilde{\mathcal{T}}. \tag{185b}$$

The changes of the period 1 steady-state values $\tilde{K}_1$ and $\tilde{B}_1$ compared to their initial (given) values $\tilde{K}_0$ and $\tilde{B}_0$ are given by :

$$\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} > 0, \tag{186a}$$

$$\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} + B_{G^T} \gtrless 0, \tag{186b}$$

where $K_{\bar{\lambda}} > 0$, $B_{\bar{\lambda}} < 0$ and $B_{G^T} > 0$. The sign of (186b) is indeterminate but we are able to determine the length of fiscal shock, denoted by $\bar{\mathcal{T}}$, for which the steady-state change (186b) is equal to zero:

$$\bar{\mathcal{T}} = -\frac{1}{r^\star} \ln \left[\frac{B_{\bar{\lambda}}\lambda_{G^T} + B_{G^T}}{B_{\bar{\lambda}}\lambda_{G^T}} \right]. \tag{187}$$

The existence of time $\bar{\mathcal{T}}$ relies upon inequality $B_{\bar{\lambda}}\lambda_{G^T} < B_{\bar{\lambda}}\lambda_{G^T} + B_{G^T} < 0$ which in turn implies that the term in square brackets is positive and less than unity. Consequently, we get the following inequality:

$$\tilde{B}_1 \lesseqgtr B_0 \quad \text{depending on whether} \quad \mathcal{T} \gtreqless \bar{\mathcal{T}}. \tag{188}$$

From (177b)-(177c), (182b)-(182c), (185) and (186a)-(186b), we are able to deduce the following inequalities:

$$K_0 < \tilde{K}_1 = \tilde{K}_{temp} < \tilde{K}_{perm}, \tag{189a}$$

$$\tilde{B}_{perm} < \tilde{B}_{temp} < B_0 \quad \text{if} \quad \mathcal{T} < \tilde{\mathcal{T}}, \tag{189b}$$

$$\tilde{B}_{temp} < \tilde{B}_{perm} < \tilde{B}_0 \quad \text{if} \quad \mathcal{T} > \tilde{\mathcal{T}}, \tag{189c}$$

where we assume that $\tilde{\mathcal{T}} < \bar{\mathcal{T}}$.

H.2 Transitional Dynamics after a Permanent Increase in G^T

As shown previously, the stable adjustment of the economy is described by a saddle-path in (K, P)-space. The capital stock, the relative price of the non-traded good, and the stock of traded bonds evolve according to:

$$K(t) = \tilde{K} + B_1 e^{\mu_1 t}, \tag{190a}$$
$$P(t) = \tilde{P} + \omega_2^1 B_1 e^{\mu_1 t}, \tag{190b}$$
$$B(t) = \tilde{B} + \Phi_1 B_1 e^{\mu_1 t}, \tag{190c}$$

where $\omega_2^1 = 0$ if $k^T > k^N$ and with

$$B_1 = K_0 - \tilde{K} = -\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\mathrm{d}G^T,$$

where we made use of the constancy of K at time $t = 0$ (i. e. K_0 is predetermined).

Case $k^N > k^T$

Using the fact that the steady-state value of the relative price of the non-traded good remains affected by a permanent rise in G^T, the initial jump of P is given by

$$\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^T}\right|_{perm} = -\omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} < 0. \tag{191}$$

From the short run static solutions, and by substituting the change in the equilibrium value of the marginal utility of wealth and the initial jump of P, we get the response of real consumption at time $t = 0$:

$$\begin{aligned}\left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^T}\right|_{perm} &= C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} + C_P \left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^T}\right|_{perm} = -\frac{\left[1 + \alpha_C \frac{1}{\mu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right]}{P_C\left[1 + \alpha_C \frac{r^\star}{(\mu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right]}, \\ &= \left[1 + \alpha_C \frac{1}{\mu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right] \left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T}\right|_{perm} < 0,\end{aligned} \tag{192}$$

where $0 < \left[1 + \alpha_C \frac{1}{\mu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right] < 1$. Therefore, we deduce the following inequality

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T}\right|_{perm} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < \left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^T}\right|_{perm} < 0. \tag{193}$$

Irrespective of sectoral capital intensities, a rise in G^T induces a once-for-all upward jump of the marginal utility of wealth which reduces real consumption. If $k^N > k^T$, the initial fall of C is moderated by the depreciation in P at time $t = 0$ and falls by less than in the long run.

Differentiating solutions (190), with respect to time, one obtains:

$$\dot{K}(t) = -\mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} e^{\mu_1 t}\mathrm{d}G^T < 0, \tag{194a}$$
$$\dot{P}(t) = -\mu_1 \omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} e^{\mu_1 t}\mathrm{d}G^T > 0, \tag{194b}$$
$$\dot{B}(t) = -\mu_1 \Phi_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} e^{\mu_1 t}\mathrm{d}G^T > 0, \tag{194c}$$

where $\Phi_1 < 0$ and $\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} < 0$.

Along the stable adjustment, real consumption decreases:

$$\dot{C} = -\sigma_C C \alpha_C \frac{\dot{P}}{P} < 0, \tag{195}$$

where $\left(r^\star - \alpha_C \frac{\dot{P}}{P} \right)$ corresponds to the consumption-based real interest rate. After its initial depreciation, the relative price of the non-traded good appreciates to revert back to its initial value. This appreciation lowers the consumption-based real interest rate below the world interest rate which stimulates real consumption.

Case $k^T > k^N$

Differentiating solutions (190), with respect to time, one obtains

$$\begin{aligned}
\dot{K}(t) &= -\mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} e^{\mu_1 t} \mathrm{d}G^T > 0, & \text{(196a)} \\
\dot{P}(t) &= 0, & \text{(196b)} \\
\dot{B}(t) &= -\mu_1 \Phi_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} e^{\mu_1 t} \mathrm{d}G^T < 0, & \text{(196c)}
\end{aligned}$$

where $\Phi_1 < 0$ and $\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} > 0$.

H.3 Transitional Dynamics after a Temporary Increase in G^T

Case $k^N > k^T$

By evaluating formal solution for $P(t)$ and differentiating with respect to G^T, we get the initial jump of P

$$\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^T}\right|_{temp} = -\omega_2^1 \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} < 0. \tag{197}$$

By adopting a similar procedure, we obtain the initial response of the investment flow following a temporary rise in government spending on the traded good :

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^T}\right|_{temp} = \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^T}\right|_{perm} < 0. \tag{198}$$

By differentiating the formal solution (104c) over period 1 for $B(t)$ with respect to time, remembering that $B_2/\mathrm{d}G^T = 0$, then evaluating this at $t = 0$, and differentiating with respect to G^T, we obtain the initial response of the current account following a fiscal expansion:

$$\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^T}\right|_{temp} = -r^\star \left[\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} - \Phi_1 \left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp} \right] + \mu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^T}.$$

The expression in brackets can be evaluated by using properties (102), and the fact that $B_{G^T} = -\lambda_{G^T}/\lambda_B$:

$$\begin{aligned}
-\left[\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} - \Phi_1 \left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp} \right] &= -\left[B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} + B_{G^T} - \Phi_1 K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} \right], \\
&= -\left[\frac{\lambda_{G^T}}{\lambda_B} \left(1 - e^{-r^\star \mathcal{T}}\right) - \frac{\lambda_{G^T}}{\lambda_B} \right], \\
&= -B_{G^T} e^{-r^\star \mathcal{T}}. \qquad (199)
\end{aligned}$$

Inserting this expression, and remembering that $\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T} = \Phi_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}$, we obtain the reaction of the current account at time $t = 0$:

$$\begin{aligned} \left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^T}\right|_{temp} &= -e^{-r^\star \mathcal{T}} - \mu_1 \Phi_1 K_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \lambda_{G^T}, \\ &= -e^{-r^\star \mathcal{T}} - \mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) \lessgtr 0. \end{aligned} \tag{200}$$

The initial current account response is the result of two conflictory forces: (i) a *smoothing* effect which deteriorates the current account, and (ii) the negative investment flow which improves the external asset position. From (200), there exists a critical value of shock's length, $\hat{\mathcal{T}} > 0$, such that the current account response is zero on impact, i. e. $\dot{B}(0) = 0$. Solving (200) for $\hat{\mathcal{T}}$, we get:

$$\hat{\mathcal{T}} = \frac{1}{r^\star} \ln \left[\frac{1 - \mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm}}{-\mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm}} \right], \tag{201}$$

where the term in square brackets is higher than one.

The dynamics for K and P over period 1 are derived by taking the time derivative of equations (104a) and (104b):

$$\dot{K}(t) = \mu_1 \frac{B_1}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T = -\mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) e^{\mu_1 t} \mathrm{d}G^T < 0, \tag{202a}$$

$$\dot{P}(t) = \mu_1 \omega_2^1 \frac{B_1}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T = -\omega_2^1 \mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) e^{\mu_1 t} \mathrm{d}G^T > 0, \tag{202b}$$

where we used the fact that $B_1/\mathrm{d}G^T = -\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right)$.

While the P and K go in the same direction as after a permanent rise in G^T, differentiation with respect to time of eq. (104c) shows that the current account may change of sign over period 1:

$$CA(t) = \dot{B}(t) = -e^{-r^\star (\mathcal{T} - t)} \mathrm{d}G^T - \mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) e^{\mu_1 t} \mathrm{d}G^T \lessgtr 0. \tag{203}$$

We have now to determine the conditions under which the current account dynamics displays a non monotonic behavior. Equation (203) reveals that the stock of international assets reaches a turning point during its transitional adjustment at time $\hat{\mathcal{T}}$ given by

$$\hat{\mathcal{T}} = \frac{1}{\mu_2} \ln \left[-\mu_1 \frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\Big|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) e^{r^\star \mathcal{T}} \right]. \tag{204}$$

The necessary condition for $\hat{\mathcal{T}} > 0$, corresponds to:

$$0 < e^{-r^\star \mathcal{T}} < -\mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) \quad \Leftrightarrow \quad \left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^T}\right|_{temp} > 0. \tag{205}$$

If the fiscal expansion lasts a short period, i. e. $\mathcal{T} < \hat{\mathcal{T}}$, the current account initially deteriorates and the stock of foreign assets decreases monotonically until time $\mathcal{T}$. If the fiscal expansion lasts a time period longer than $\hat{\mathcal{T}}$, the current account initially improves before reaching a turning point at time $\hat{\mathcal{T}}$. Subsequently, the current account deteriorates until time $\mathcal{T}$.

Once the government policy has been removed at time $\mathcal{T}$, the relative price of the non-traded good keeps on depreciating and the capital stock converges towards its new lower steady-state value:

$$\begin{aligned} \dot{K}(t) &= \mu_1 \frac{B_1'}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T < 0, & (206a) \\ \dot{P}(t) &= \mu_1 \omega_2^1 \frac{B_1'}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T > 0, & (206b) \end{aligned}$$

where $B_1'/\mathrm{d}G^T = B_1/\mathrm{d}G^T > 0$. Over period 2, the current account improves unambiguously as it can be seen from the time derivative of solution (106c):

$$\dot{B}(t) = \mu_1 \Phi_1 \frac{B_1'}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T > 0. \tag{207}$$

Case $k^T > k^N$

If $k^T > k^N$, the dynamics for P are flat as after a permanent fiscal expansion since the constant $B_2/\mathrm{d}G^T$ is zero, i.e. $\dot{P}(t) = 0$. The investment flow is positive over period 1

$$I(t) = \dot{K}(t) = \mu_1 \frac{B_1}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T = -\mu_1 K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} e^{\mu_1 t} \mathrm{d}G^T > 0. \tag{208}$$

Differentiating eq. (104c) with respect to time and remembering that $B_2/\mathrm{d}G^T = 0$ yields the transitional path for $B(t)$:

$$CA(t) = -r^\star \left[\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} - \Phi_1 \left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp} \right] e^{r^\star t} \mathrm{d}G^T + \mu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T. \tag{209}$$

By evaluating this expressions at $t = 0$, and differentiating with respect to G^T, we obtain the initial response of the current account following a fiscal expansion:

$$\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^T}\right|_{temp} = -r^\star \left[\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} - \Phi_1 \left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp} \right] + \mu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^T}.$$

The expression in brackets can be evaluated by using properties (102), and the fact that $B_{G^T} = -\lambda_{G^T}/\lambda_B$:

$$\begin{aligned} -\left[\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} - \Phi_1 \left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp} \right] &= -\left[B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} + B_{G^T} - \Phi_1 K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} \right], \\ &= -\left[\frac{\lambda_{G^T}}{\lambda_B} \left(1 - e^{-r^\star \mathcal{T}}\right) - \frac{\lambda_{G^T}}{\lambda_B} \right], \\ &= -B_{G^T} e^{-r^\star \mathcal{T}}. \end{aligned} \tag{210}$$

Inserting expression (210) and remembering that $\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T} = \Phi_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}$, we obtain the reaction of the current account at time $t = 0$:

$$\begin{aligned} \left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^T}\right|_{temp} &= -e^{-r^\star \mathcal{T}} - \mu_1 \Phi_1 K_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \lambda_{G^T}, \\ &= -\left[e^{-r^\star \mathcal{T}} + \mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) \right] < 0. \end{aligned} \tag{211}$$

If $k^T > k^N$, both the *smoothing* effect and the positive investment flow lead to a decumulation of foreign assets. Consequently, the current account deteriorates initially and the stock of internationally traded bonds keeps on decreasing over period 1:

$$CA(t) = \dot{B}(t) = -e^{-r^\star(\mathcal{T}-t)}\mathrm{d}G^T - \mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) e^{\mu_1 t}\mathrm{d}G^T < 0. \tag{212}$$

Over period 2, the stocks of physical capital keeps on decreasing and the current account deteriorates monotonically:

$$I(t) = \mu_1 \frac{B_1'}{\mathrm{d}G^T} e^{\mu_1 t}\mathrm{d}G^T > 0, \tag{213a}$$

$$CA(t) = \mu_1 \Phi_1 \frac{B_1'}{\mathrm{d}G^T} e^{\mu_1 t}\mathrm{d}G^T < 0, \tag{213b}$$

where $B_1'/\mathrm{d}G^T = B_1/\mathrm{d}G^T < 0$.

I The Effects of Temporary Fiscal Shocks: The Case of Elastic Labor Supply

In this section, we derive formal solutions by assuming elastic labor supply. We consider a traded sector alternatively more or less capital intensive than the non-traded sector.

We first solve the system (72a)-(72c) for $\tilde{P}$, $\tilde{K}$ and $\tilde{B}$ as functions of the marginal utility of wealth, $\bar{\lambda}$ and government spending G^N. Totally differentiating equations (72a)-(72c) yields in matrix form:

$$\begin{pmatrix} \frac{h_{kk}k_P^N}{\mu} & 0 & 0 \\ \left(\frac{Y_P^N}{\mu} - C_P^N\right) & \left(\frac{Y_K^N}{\mu} - \delta_K\right) & 0 \\ \left(Y_P^T - C_P^T\right) & Y_K^T & r^\star \end{pmatrix} \begin{pmatrix} \mathrm{d}\tilde{P} \\ \mathrm{d}\tilde{K} \\ \mathrm{d}\tilde{B} \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ -\left(\frac{Y_{\bar{\lambda}}^N}{\mu} - C_{\bar{\lambda}}^N\right)\mathrm{d}\bar{\lambda} + \mathrm{d}G^N \\ -\left(Y_{\bar{\lambda}}^T - C_{\bar{\lambda}}^T\right)\mathrm{d}\bar{\lambda} \end{pmatrix}. \tag{214}$$

Steady-state values of K and B can be expressed as functions of the shadow value of wealth and government spending G^N:

$$\tilde{K} = K\left(\bar{\lambda}, G^N\right), \tag{215a}$$

$$\tilde{B} = B\left(\bar{\lambda}, G^N\right), \tag{215b}$$

with partial derivatives given by:

$$K_{\bar{\lambda}} \equiv \frac{\partial \tilde{K}}{\partial \bar{\lambda}} = -\frac{\left(\frac{Y_{\bar{\lambda}}^N}{\mu} - C_{\bar{\lambda}}^N\right)}{\left(\frac{Y_K^N}{\mu} - \delta_K\right)}, \tag{216a}$$

$$B_{\bar{\lambda}} \equiv \frac{\partial \tilde{B}}{\partial \bar{\lambda}} = \frac{Y_K^T\left(\frac{Y_{\bar{\lambda}}^N}{\mu} - C_{\bar{\lambda}}^N\right) - \left(\frac{Y_K^N}{\mu} - \delta_K\right)\left(Y_{\bar{\lambda}}^T - C_{\bar{\lambda}}^T\right)}{r^\star\left(\frac{Y_K^N}{\mu} - \delta_K\right)}. \tag{216b}$$

(216c)

We sign expressions depending on whether the traded sector is more or less capital intensive than the non-traded sector:

$$K_{\bar{\lambda}} \equiv \frac{\partial \tilde{K}}{\partial \bar{\lambda}} = -\frac{1}{\bar{\lambda}}\frac{1}{\nu_1}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_1+\delta_K\right)\right] > 0 \quad \text{if} \quad k^T > k^N, \tag{217a}$$

$$= -\frac{1}{\bar{\lambda}}\frac{1}{\nu_2}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2+\delta_K\right)\right] \gtrless 0 \quad \text{if} \quad k^N > k^T, \tag{217b}$$

$$B_{\bar{\lambda}} \equiv \frac{\partial \tilde{B}}{\partial \bar{\lambda}} = \frac{\left\{\sigma_C\left(\tilde{P}\tilde{C}^N\nu_2 - \tilde{C}^T\nu_1\right) + \nu_2\tilde{P}\sigma_L\tilde{L}\left[\tilde{k}^N\nu_1 - \tilde{k}^T\left(\nu_1+\delta_K\right)\right]\right\}}{r^\star \nu_1 \bar{\lambda}} < 0, \quad \text{if} \quad k^T > k^N, \tag{217c}$$

$$= \frac{\left\{\sigma_C\left(\tilde{P}\tilde{C}^N\nu_1 - \tilde{C}^T\nu_2\right) + \nu_1\tilde{P}\sigma_L\tilde{L}\left[\tilde{k}^N\nu_2 - \tilde{k}^T\left(\nu_2+\delta_K\right)\right]\right\}}{r^\star \nu_2 \bar{\lambda}} < 0, \quad \text{if} \quad k^N > k^T, \tag{217d}$$

and

$$K_{G^N} \equiv \frac{\partial \tilde{K}}{\partial G^N} = \frac{1}{\frac{Y_K^N}{\mu} - \delta_K} = \frac{1}{\nu_1} < 0 \quad \text{if} \quad k^T > k^N, \tag{218a}$$

$$= \frac{1}{\frac{Y_K^N}{\mu} - \delta_K} = \frac{1}{\nu_2} > 0 \quad \text{if} \quad k^N > k^T, \tag{218b}$$

$$B_{G^N} \equiv \frac{\partial \tilde{B}}{\partial G^N} = -\frac{Y_K^T}{\left(\frac{Y_K^N}{\mu} - \delta_K\right) r^\star} = -\frac{\tilde{P}\nu_2}{r^\star\nu_1} > 0, \quad \text{if} \quad k^T > k^N, \tag{218c}$$

$$= -\frac{Y_K^T}{\left(\frac{Y_K^N}{\mu} - \delta_K\right) r^\star} = -\frac{\tilde{P}\nu_1}{r^\star\nu_2} > 0, \quad \text{if} \quad k^N > k^T. \tag{218d}$$

(218e)

Adopting the same procedure as described in section K.7, we derive formal expressions below for constants B_1, B_2 and B_1' when $k^T > k^N$. We were unable to derive useful formal expressions with the reversal of capital intensities. Yet, in the latter case, analytical results derived by assuming inelastic labor supply are in line with numerical results and thereby elastic labor supply does not affect qualitatively the results.

Case $k^T > k^N$

The solutions after a rise in G^N are:

$$\frac{B_1}{\mathrm{d}G^N} = -\frac{\left[\sigma_C\left(\tilde{P}\tilde{C}^N e^{-r^\star \mathcal{T}} + \tilde{C}^T\right) - \sigma_L\tilde{L}\tilde{P}\left(\nu_2\tilde{k}^N + \left(\nu_1+\delta_K\right)\tilde{k}^T e^{-r^\star\mathcal{T}}\right)\right]}{\nu_1\left(\sigma_C P_C\tilde{C} + \sigma_L\tilde{W}\tilde{L}\right)},$$

$$= -\frac{\left[\left(\sigma_C P_C\tilde{C} + \sigma_L\tilde{W}\tilde{L}\right) + \left(1-e^{-r^\star\mathcal{T}}\right)\tilde{P}\left(\sigma_L\tilde{L}\tilde{k}^T\left(\nu_1+\delta_K\right) - \sigma_C\tilde{C}^N\right)\right]}{\nu_1\left(\sigma_C P_C\tilde{C} + \sigma_L\tilde{W}\tilde{L}\right)} \gtrless 0, \tag{219a}$$

$$\frac{B_2}{\mathrm{d}G^N} = 0, \tag{219b}$$

$$\frac{B_1'}{\mathrm{d}G^N} = \frac{B_1}{\mathrm{d}G^N} + K_{G^N}e^{-\nu_1\mathcal{T}}$$

$$= -\frac{\left[\left(\sigma_C P_C\tilde{C} + \sigma_L\tilde{W}\tilde{L}\right)\left(1-e^{-\nu_1\mathcal{T}}\right) + \left(1-e^{-r^\star\mathcal{T}}\right)\tilde{P}\left(\sigma_L\tilde{L}\tilde{k}^T\left(\nu_1+\delta_K\right) - \sigma_C\tilde{C}^N\right)\right]}{\nu_1\left(\sigma_C P_C\tilde{C} + \sigma_L\tilde{W}\tilde{L}\right)} \gtrless 0, \tag{219c}$$

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \lambda_{G^N}\left(1-e^{-r^\star\mathcal{T}}\right) > 0, \tag{219d}$$

where λ_{G^N} represents the change in the equilibrium value of the shadow value of wealth after a permanent increase in G^N (see eq. (79b)).

General solutions for K and P are:

$$K(t) - \tilde{K} = B_1 e^{\nu_1 t} + B_2 e^{\nu_2 t}, \tag{220a}$$
$$P(t) - \tilde{P} = \omega_2^1 B_1 e^{\nu_1 t} + \omega_2^2 B_2 e^{\nu_2 t}, \tag{220b}$$

Differentiating eq. (220a) w.r.t. time, evaluating at time $t = 0$ and differentiating w.r.t. G^N, we obtain the initial response of investment following a temporary rise in government spending on the non-traded good:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} = \nu_1 \frac{B_1}{\mathrm{d}G^N} + \nu_2 \frac{B_2}{\mathrm{d}G^N}.$$

Substituting (219a) and using the fact that $\frac{B_2}{\mathrm{d}G^N} = 0$, the initial reaction of investment is:

$$\begin{aligned}
\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} &= -\nu_1 \frac{\left[\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right) + \left(1 - e^{-r^\star \mathcal{T}}\right) \tilde{P} \left(\sigma_L \tilde{L}\tilde{k}^T \left(\nu_1 + \delta_K\right) - \sigma_C \tilde{C}^N\right)\right]}{\nu_1 \left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)}, \\
&= -\left[1 + \left(1 - e^{-r^\star \mathcal{T}}\right) \frac{\left[\sigma_L \tilde{L}\tilde{k}^T \tilde{P} \left(\nu_1 + \delta_K\right) - \sigma_C \tilde{P}\tilde{C}^N\right]}{\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)}\right] \lesseqgtr 0.
\end{aligned} \tag{221}$$

Eq. (221) corresponds to **eq. (24) in the text**. Since the length of the shock $\mathcal{T}$ plays a key role in driving the initial response of investment, it is useful to determine the critical length $\hat{\mathcal{T}}$ such that when $\mathcal{T} < \hat{\mathcal{T}}$, government spending crowds out investment. Investment falls when

$$\begin{aligned}
&-\left[1 + \left(1 - e^{-r^\star \mathcal{T}}\right) \frac{\left(\sigma_L \tilde{L}\tilde{k}^T \tilde{P} \left(\nu_1 + \delta_K\right) - \sigma_C \tilde{P}\tilde{C}^N\right)}{\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)}\right] < 0, \\
&e^{-r^\star \mathcal{T}} > 1 + \frac{\left[\sigma_L \tilde{L}\tilde{k}^T \tilde{P} \left(\nu_1 + \delta_K\right) - \sigma_C \tilde{P}\tilde{C}^N\right]}{\left[\sigma_L \tilde{L}\tilde{k}^T \tilde{P} \left(\nu_1 + \delta_K\right) - \sigma_C \tilde{P}\tilde{C}^N\right]}, \\
&\mathcal{T} < -\frac{1}{r^\star} \ln \left[\frac{\left(\sigma_C \tilde{C}^T - \sigma_L \tilde{L}\tilde{k}^N \tilde{P}\nu_2\right)}{\left(\sigma_L \tilde{L}\tilde{k}^T \tilde{P} \left(\nu_1 + \delta_K\right) - \sigma_C \tilde{P}\tilde{C}^N\right)}\right] = \hat{\mathcal{T}},
\end{aligned}$$

where we used the fact that $\tilde{W} + \tilde{P}\tilde{k}^T \left(\nu_1 + \delta_K\right) = -\tilde{P}\tilde{k}^N \nu_2$. The term in brackets in positive but smaller than one. When $\mathcal{T}$ is smaller than the critical length $\hat{\mathcal{T}}$, then investment is crowded-out by public spending.

The general solution for the stock of foreign assets is given by:

$$B(t) = \tilde{B} + \left[\left(B_0 - \tilde{B}\right) - \Phi_1 B_1 - \Phi_2 B_2\right] e^{r^\star t} + \Phi_1 B_1 e^{\nu_1 t} + \Phi_2 B_2 e^{\nu_2 t}, \tag{222}$$

Differentiating eq. (222) w.r.t. time, evaluating at time $t = 0$ and differentiating w.r.t. G^N, we obtain the initial response of the current account after a temporary rise in G^N:

$$\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} = r^\star \left[-\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \Phi_1 \frac{B_1}{\mathrm{d}G^N} - \Phi_2 \frac{B_2}{\mathrm{d}G^N}\right] + \nu_1 \frac{B_1 \Phi_1}{\mathrm{d}G^N} + \nu_2 \frac{B_2 \Phi_2}{\mathrm{d}G^N}.$$

Using the fact that

$$
\begin{aligned}
& -\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \Phi_1 \frac{B_1}{\mathrm{d}G^N} - \Phi_2 \frac{B_2}{\mathrm{d}G^N} \\
= & -\left[\left(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + \left(B_{G^N} - \Phi_1 K_{G^N}\right)\right], \\
= & \frac{\lambda_{G^N}}{\lambda_B} e^{-r^\star \mathcal{T}} = -\frac{\tilde{P}}{r^\star} e^{-r^\star \mathcal{T}},
\end{aligned} \tag{223}
$$

the initial reaction of the current account can be rewritten as follows:

$$
\begin{aligned}
\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} &= -\tilde{P} e^{-r^\star \mathcal{T}} - \nu_1 \tilde{P} \frac{B_1}{\mathrm{d}G^N}, \\
&= \tilde{P}\left(1 - e^{-r^\star \mathcal{T}}\right)\left[1 + \frac{\left(\sigma_L \tilde{L} \tilde{k}^T \tilde{P}\left(\nu_1 + \delta_K\right) - \sigma_C \tilde{P} \tilde{C}^N\right)}{\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W} \tilde{L}\right)}\right] \lessgtr 0, \\
&= -\tilde{P} e^{-r^\star \mathcal{T}} + \tilde{P}\left[1 + \left(1 - e^{-r^\star \mathcal{T}}\right) \frac{\left[\sigma_L \tilde{L} \tilde{k}^T \tilde{P}\left(\nu_1 + \delta_K\right) - \sigma_C \tilde{P} \tilde{C}^N\right]}{\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W} \tilde{L}\right)}\right] \lessgtr 0,
\end{aligned} \tag{224}
$$

where we used the fact that $\Phi_1 = -\tilde{P}$. Eq. (224) corresponds to **eq. (25) in the text**.

Case $k^N > k^T$

While we are unable to derive full expressions for temporary shocks if the non traded sector is more capital intensive than the traded sector when considering elastic labor supply, we are able to provide useful (i.e., interpretable) expressions which are included and discussed in the text. Below, we provide details about the derivations of these useful expressions.

The solutions after a rise in G^N are:

$$
\frac{B_1}{\mathrm{d}G^N} = -\frac{B_2}{\mathrm{d}G^N} - K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - K_{G^N} = -\frac{\left(1 - e^{-\nu_2 T}\right)}{\nu_2} - K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp}, \tag{225a}
$$

$$
\frac{B_2}{\mathrm{d}G^N} = -\frac{e^{-\nu_2 T}}{\nu_2}, \tag{225b}
$$

$$
\frac{B_1'}{\mathrm{d}G^N} = \frac{B_1}{\mathrm{d}G^N}, \tag{225c}
$$

$$
\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \lambda_{G^N}\left(1 - e^{-r^\star \mathcal{T}}\right) + \frac{\left(\Phi_1 - \Phi_2\right)\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 T}\right)}{\nu_2\left(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right)} > 0, \tag{225d}
$$

where we computed the following relationship to sign (225d)

$$
\Phi_1 - \Phi_2 = -\frac{\omega_2^1}{\nu_2}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T\left(\nu_2 + \delta_K\right)\right] < 0. \tag{226}
$$

The sign of (226) holds when labor supply is elastic enough (i.e., for plausible values of σ_L).

Using the fact that $\frac{1}{\left(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right)} = \frac{\lambda_{G^N}}{\left(B_{G^N} - \Phi_1 K_{G^N}\right)}$, eq. (225d) can be rewritten as follows:

$$
\begin{aligned}
\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} &= \lambda_{G^N}\left\{\left(1 - e^{-r^\star \mathcal{T}}\right) - \frac{\left(\Phi_1 - \Phi_2\right)\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 T}\right)}{\nu_2\left(B_{G^N} - \Phi_1 K_{G^N}\right)}\right\} > 0, \\
&= \lambda_{G^N}\left\{\left(1 - e^{-r^\star \mathcal{T}}\right) - \frac{r^\star\left(\Phi_1 - \Phi_2\right)\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 T}\right)}{\nu_2 \tilde{P} - r^\star\left(\Phi_1 - \Phi_2\right)}\right\} > 0,
\end{aligned} \tag{227}
$$

where $B_{G^N} - \Phi_1 K_{G^N}$ is given by

$$\begin{aligned} B_{G^N} - \Phi_1 K_{G^N} &= \frac{\tilde{P}}{r^\star} + \frac{\omega_2^1}{(\nu_2)^2}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\right], \\ &= \frac{\tilde{P}}{r^\star} - \frac{\Phi_1 - \Phi_2}{\nu_2} > 0. \end{aligned}$$

To derive a more easily interpretable expression for the initial reaction of investment after a temporary rise in G^N, we proceed as in section G. Hence, we first linearize the non-traded good market clearing condition in the neighborhood of the steady-state:

$$I(t) - \tilde{I} = \frac{Y_K^N}{\mu}\left(K(t) - \tilde{K}\right) + \left(\frac{Y_P^N}{\mu} - C_P^N\right)\left(P(t) - \tilde{P}\right).$$

Using the fact that $\mathrm{d}\tilde{I} = \frac{Y_K^N}{\mu}\mathrm{d}\tilde{K} + \left(\frac{Y_P^N}{\mu} - C_P^N\right)\mathrm{d}\tilde{P} + \left(\frac{Y_{\bar{\lambda}}^N}{\mu} - C_{\bar{\lambda}}^N\right)\mathrm{d}\bar{\lambda}\big|_{temp} - \mathrm{d}G^N$, and evaluating the above expression at time $t = 0$, we get:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} = \left(\frac{Y_P^N}{\mu} - C_P^N\right)\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{temp} + \frac{\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\right]}{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - 1. \quad (228)$$

Using the fact that $\mathrm{d}\tilde{P} = 0$, we evaluate the initial jump of P which is given by:

$$\begin{aligned} \left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{temp} &= \omega_2^1 \frac{\mathrm{d}B_1}{\mathrm{d}G^N} = -\omega_2^1\left[K_{G^N}\left(1 - e^{-\nu_2 \mathcal{T}}\right) + K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp}\right], \\ &= \omega_2^1\left[-\frac{\left(1 - e^{-\nu_2 \mathcal{T}}\right)}{\nu_2} + \frac{\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\right]}{\bar{\lambda}\nu_2}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp}\right], \quad (229) \end{aligned}$$

where we substituted $K_{G^N} = 1/\nu_2$ and $K_{\bar{\lambda}} = -\frac{1}{\bar{\lambda}}\frac{1}{\nu_2}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\right]$ (see (217b)). Substituting (229) into (228), using the fact that $\omega_2^1 = \frac{\nu_1 - \nu_2}{\left(\frac{Y_P^N}{\mu} - C_P^N\right)}$, and collecting terms, the initial reaction of investment can be rewritten as:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} = \left(\frac{\nu_2 - \nu_1}{\nu_2}\right)\left(1 - e^{-\nu_2 \mathcal{T}}\right) + \frac{\nu_1}{\nu_2}\frac{\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\right]}{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - 1. \quad (230)$$

By differentiating the formal solution for foreign assets over period 1 for $B(t)$ with respect to time, then evaluating the resulting expression at $t = 0$, and differentiating with respect to G^N, we obtain the initial response of the current account following a temporary fiscal expansion:

$$\begin{aligned} \left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} &= r^\star\left\{-\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \Phi_1 \frac{B_1}{\mathrm{d}G^N} - \Phi_2 \frac{B_2}{\mathrm{d}G^N}\right\} \\ &\quad + \nu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^N} + \nu_2 \Phi_2 \frac{B_2}{\mathrm{d}G^N}. \quad (231) \end{aligned}$$

In order to simplify the solution (231), we rewrite the term in square brackets as follows

$$\begin{aligned}
& -\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \left[\Phi_1 \frac{B_1}{\mathrm{d}G^N} + \Phi_2 \frac{B_2}{\mathrm{d}G^N}\right] \\
= & -\left(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - \left(B_{G^N} - \Phi_1 K_{G^N}\right) + \left[\Phi_1 - \Phi_2\right] \frac{B_2}{\mathrm{d}G^N}, \\
= & -\left(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right) \left\{ \lambda_{G^N} \left(1 - e^{-r^\star \mathcal{T}}\right) + \frac{\left(\Phi_1 - \Phi_2\right)\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 T}\right)}{\nu_2 \left(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right)} \right\} \\
& -\left(B_{G^N} - \Phi_1 K_{G^N}\right) - \frac{\left(\Phi_1 - \Phi_2\right)}{\nu_2} e^{-\nu_2 T}, \\
= & -\frac{\left(\Phi_1 - \Phi_2\right)}{\nu_2} e^{-r^\star \mathcal{T}} - \left(B_{G^N} - \Phi_1 K_{G^N}\right) e^{-r^\star \mathcal{T}}, \\
= & -\frac{\tilde{P}}{r^\star} e^{-r^\star \mathcal{T}} < 0,
\end{aligned} \tag{232}$$

where we have computed the following expression to get (232):

$$\left(B_{G^N} - \Phi_1 K_{G^N}\right) = \frac{\tilde{P}}{r^\star} + \frac{\omega_2^1}{\left(\nu_2\right)^2} \left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \left(\nu_2 + \delta_K\right)\right] > 0. \tag{233}$$

Inserting (232) into (231), the initial response of the current account can be rewritten as follows:

$$\begin{aligned}
\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} & = -\tilde{P} e^{-r^\star \mathcal{T}} - \nu_1 \left\{ \tilde{P} + \frac{\omega_2^1}{\nu_2} \left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \left(\nu_2 + \delta_K\right)\right] \right\} \frac{B_1}{\mathrm{d}G^N} + \nu_2 \Phi_2 \frac{B_2}{\mathrm{d}G^N}, \\
& = -\tilde{P} e^{-r^\star \mathcal{T}} - \frac{\nu_1}{\nu_2} \left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \left(\nu_2 + \delta_K\right)\right] \omega_2^1 \frac{B_1}{\mathrm{d}G^N} - \tilde{P} \left(\nu_1 \frac{B_1}{\mathrm{d}G^N} + \nu_2 \frac{B_2}{\mathrm{d}G^N}\right), \\
& = -\tilde{P} e^{-r^\star \mathcal{T}} - \frac{\nu_1}{\nu_2} \left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \left(\nu_2 + \delta_K\right)\right] \left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{temp} - \tilde{P} \left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp},
\end{aligned} \tag{234}$$

where $\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\big|_{temp}$ is given by (229) and $\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\big|_{temp}$ is given by (230). To get (234), we used the fact that $\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\big|_{temp} = \nu_1 \frac{B_1}{\mathrm{d}G^N} + \nu_2 \frac{B_2}{\mathrm{d}G^N}$.

J Savings

Since the current account can be alternatively expressed as net exports plus interest earnings from traded bond holding, or as the savings less investment, we provide details for the derivation of steady-state and dynamic effects of fiscal shocks on savings.

J.1 Formal Solution for Financial Wealth

The law of motion for financial wealth ($S(t) = \dot{A}(t)$) is given by:

$$\dot{A}(t) = r^\star A(t) + W(P) L\left(\bar{\lambda}, P\right) - P_C(P) C\left(\bar{\lambda}, P\right) - Z, \tag{235}$$

with $Z = G^T + P G^N$.

The linearized version of (235) is:

$$\dot{A}(t) = r^\star \left(A(t) - \tilde{A}\right) + M\left(P(t) - \tilde{P}\right), \tag{236}$$

with M given by

$$\begin{aligned} M &= \left(W_P \tilde{L} + \tilde{W} L_P\right) - \left(\tilde{C}^N + P_C C_P + G^N\right), \\ &= \tilde{L} W_P \left(1 + \sigma_L\right) - \left[\tilde{C}^N \left(1 - \sigma_C\right) + G^N\right], \\ &= -\left\{\tilde{K}\left(\nu_2 + \delta_K\right) + \left[\sigma_L \tilde{L} \tilde{k}^T \left(\nu_2 + \delta_K\right) - \sigma_C \tilde{C}^N\right]\right\} < 0. \end{aligned} \tag{237}$$

From the second line of (237), if $\sigma_C < 1$ as empirical studies suggest, then the term in square brackets is positive and M is negative. The last line has been computed by using the fact that $\tilde{L} = \tilde{L}^N + \tilde{L}^T$ and $\tilde{K} = \tilde{k}^T \tilde{L}^T + \tilde{k}^N \tilde{L}^N$ which allows to simplify $\frac{1}{\mu}\left[\tilde{Y}^N + \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\mu\right]$ to $\tilde{K}\left(\nu_2 + \delta_K\right)$.

The general solution for the stock of financial wealth is given by:

$$A(t) = \tilde{A} + \left[\left(A_0 - \tilde{A}\right) - \frac{M\omega_2^1}{\nu_1 - r^\star} B_1 - \frac{M\omega_2^2}{\nu_2 - r^\star} B_2\right] e^{r^\star t} + \frac{M\omega_2^1}{\nu_1 - r^\star} B_1 e^{\nu_1 t} + \frac{M\omega_2^2}{\nu_2 - r^\star} B_2 e^{\nu_1 t}. \tag{238}$$

Invoking the transversality condition, we obtain the stable solution for financial wealth:

$$A(t) = \tilde{A} + \frac{M\omega_2^1}{\nu_1 - r^\star} B_1 e^{\nu_1 t}, \tag{239}$$

and the intertemporal solvency condition

$$\tilde{A} - A_0 = \frac{M\omega_2^1}{\nu_1 - r^\star}\left(\tilde{K} - K_0\right). \tag{240}$$

J.2 Steady-State and Dynamic Effects of a Permanent Fiscal Shock

Differentiating (240) w. r. t. G^i $(i = T, N)$, long-term changes of financial wealth are given by:

$$\frac{\mathrm{d}\tilde{A}}{\mathrm{d}G^i} = \frac{\omega_2^1}{\nu_2}\left(\tilde{K}\nu_2 + \sigma_L \tilde{L}\tilde{k}^T \nu_2 - \sigma_C \tilde{C}^N\right)\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^i}. \tag{241}$$

Differentiating (239) w. r. t. G^i $(i = T, N)$, we get the dynamics of savings:

$$S(t) = \dot{A}(t) = \nu_1 \frac{M\omega_2^1}{\nu_1 - r^\star}\frac{B_1}{\mathrm{d}G^i}\mathrm{d}G^i e^{\nu_1 t}, \tag{242}$$

where $\frac{B_1}{\mathrm{d}G^i} = -\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^i}$.

J.3 Steady-State and Dynamic Effects of a Temporary Fiscal Shock

We now evaluate the transitional dynamics of saving after a temporary shock, $\mathrm{d}G_i$ $(i = T, N)$.

Case $k^N > k^T$

Over the unstable period 1, savings evolve as follows:

$$S(t) = \dot{A}(t) = r^\star\left[\left(A_0 - \tilde{A}_1\right) - \frac{M\omega_2^1}{\nu_1 - r^\star}B_1\right]e^{r^\star t} + \nu_1\frac{M\omega_2^1}{\nu_1 - r^\star}B_1 e^{\nu_1 t}, \tag{243}$$

with

$$\left(A_0 - \tilde{A}_1\right) = \left(B_0 - \tilde{B}_1\right) + \tilde{P}_0\left(K_0 - \tilde{K}_1\right) + K_0\left(P_0 - \tilde{P}_1\right). \tag{244}$$

Over the stable period 2, savings evolve as follows:

$$S(t) = \dot{A}(t) = \nu_1\frac{M\omega_2^1}{\nu_1 - r^\star}B_1' e^{\nu_1 t}. \tag{245}$$

To compute steady-state changes of the stock of financial wealth, we linearize $A(t) = B(t) + P(t)K(t)$ in the neighborhood of the final steady-state. We have:

$$A(t) - \tilde{A}_2 = \left(B(t) - \tilde{B}_2\right) + \tilde{P}\left(K(t) - \tilde{K}_2\right) + \tilde{K}\left(P(t) - \tilde{P}_2\right).$$

Then we evaluate at time $t = 0$:

$$A_0 - \tilde{A}_2 = \left(B_0 - \tilde{B}_2\right) + \tilde{P}_0\left(K_0 - \tilde{K}_2\right) + \tilde{K}_0\left(P(0) - \tilde{P}_2\right),$$

where we used the fact that $A(0) = A_0$, $B(0) = B_0$, $K(0) = K_0$ and assumed that the small open economy starts initially from the steady-state, i. e. $A_0 = \tilde{A}_0 = \tilde{A}$, $B_0 = \tilde{B}_0 = \tilde{B}$, $K_0 = \tilde{K}_0 = \tilde{K}$. Substituting $P(0) - \tilde{P}_2 = \omega_2^1 B_1$ into the expression above and differentiating w.r.t G^i $(i = T, N)$, long-term changes of financial wealth are given by:

$$\left.\frac{\mathrm{d}\tilde{A}}{\mathrm{d}G^T}\right|_{temp} = \left(B_{\bar{\lambda}} + \tilde{P}K_{\bar{\lambda}}\right)\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} < 0, \tag{246a}$$

$$\left.\frac{\mathrm{d}\tilde{A}}{\mathrm{d}G^N}\right|_{temp} = \left(B_{\bar{\lambda}} + \tilde{P}K_{\bar{\lambda}}\right)\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \tag{246b}$$

with

$$\left(B_{\bar{\lambda}} + \tilde{P}K_{\bar{\lambda}}\right) = -\frac{\sigma_C P_C \tilde{C}}{\bar{\lambda} r^\star} < 0. \tag{247}$$

Case $k^T > k^N$

Since $\omega_2^1 = 0$ whenever the traded good sector is relatively more capital intensive, and because $B_2/\mathrm{d}G^i = 0$, the transitional dynamics for saving degenerate and the financial wealth jumps immediately to its new steady-state level.

Adopting a similar procedure than previously (i. e. in the case $k^N > k^T$), we can calculate the long-term changes of financial wealth as follows:

$$\left.\frac{\mathrm{d}\tilde{A}}{\mathrm{d}G^T}\right|_{temp} = \left(B_{\bar{\lambda}} + \tilde{P}K_{\bar{\lambda}}\right)\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} < 0, \tag{248a}$$

$$\left.\frac{\mathrm{d}\tilde{A}}{\mathrm{d}G^N}\right|_{temp} = \left(B_{\bar{\lambda}} + \tilde{P}K_{\bar{\lambda}}\right)\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \tag{248b}$$

with

$$B_{\bar{\lambda}} + \tilde{P}K_{\bar{\lambda}} = -\frac{\sigma_C P_C \tilde{C}}{\bar{\lambda} r^\star} < 0. \tag{249}$$

K The Case of Endogenous Markup

The framework builds on Jaimovich and Floetotto [2008]. While we consider the case of an endogenous markup, the framework is identical to that with a fixed markup, except that in the latter case the number of competitors is large enough so that the price-elasticity of demand is not affected by firm entry. There are two sectors in the economy: a perfectly competitive sector which produces a traded good denoted by the superscript T and an imperfectly competitive sector which produces a non-traded good denoted by the superscript N. We assume that each producer of a unique variety of the non-traded good has the following technology $X_j^N = H\left(\mathcal{K}_j, \mathcal{L}_j\right)$ where $\mathcal{K}_j$ is the capital stock and $\mathcal{L}_j$ is labor.

K.1 Framework

The final non-traded output, Y^N, is produced in a competitive retail sector using a constant-returns-to-scale production function which aggregates a continuum measure one of sectoral non-traded goods:

$$Y^N = \left[\int_0^1 \left(\mathcal{Q}_j^N\right)^{\frac{\omega-1}{\omega}} \mathrm{d}j\right]^{\frac{\omega}{\omega-1}}, \tag{250}$$

where $\omega > 0$ represents the elasticity of substitution between any two different sectoral goods and $\mathcal{Q}_j^N$ stands for intermediate consumption of sector'j variety (with $j \in [0, N]$). The final good producers behave competitively, and the households use the final good for both consumption and investment.

In each of the j sectors, there are $N > 1$ firms producing differentiated goods that are aggregated into a sectoral non-traded good by a CES aggregating function. The non-traded output sectoral good j is given by:[42]

$$\mathcal{Q}_j^N = N^{-\frac{1}{\epsilon-1}} \left[\int_0^N \left(\mathcal{X}_{i,j}^N\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{251}$$

where $\mathcal{X}_{i,j}^N$ stands for output of firm i in sector j and ϵ is the elasticity of substitution between any two varieties.

Denoting by P and $\mathcal{P}_j$ the relative price of the final good and of the jth variety of the intermediate good, respectively, the profit the final good producer is written as follows:

$$\Pi^N = P\left[\int_0^N \left(\mathcal{Q}_j^N\right)^{\frac{\omega-1}{\omega}} \mathrm{d}j\right]^{\frac{\omega}{\omega-1}} - \int_0^1 \mathcal{P}_j \mathcal{Q}_j^N \mathrm{d}j. \tag{252}$$

Total cost minimizing for a given level of final output gives the (intratemporal) demand function for each input:

$$\mathcal{Q}_j^N = \left(\frac{\mathcal{P}_j}{P}\right)^{-\omega} Y^N, \tag{253}$$

and the price of the final output is given by:

$$P = \left(\int_0^1 \mathcal{P}_j^{1-\omega} \mathrm{d}j\right)^{\frac{1}{1-\omega}}, \tag{254}$$

where $\mathcal{P}_j$ is the price index of sector j and P is the price of the final good.

Within each sector, there is monopolistic competition; each firm that produces one variety $\mathcal{X}_{i,j}^N$ is a price setter. Intermediate output $\mathcal{X}_{i,j}^N$ is produced using capital $\mathcal{K}_{i,j}^N$ and labor $\mathcal{L}_{i,j}^N$:

$$\mathcal{X}_{i,j}^N = H\left(\mathcal{K}_{i,j}^N, \mathcal{L}_{i,j}^N\right). \tag{255}$$

Denoting by $\mathcal{P}_{i,j}$ the price of good i in sector j, the profit function for the jth sector good producer denoted by π_j^N is:

$$\pi_j^N \equiv \mathcal{P}_j N^{-\frac{1}{\epsilon-1}} \left(\int_0^N \left(\mathcal{X}_{i,j}^N\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^N \mathcal{P}_{i,j}\mathcal{X}_{i,j}^N \mathrm{d}i. \tag{256}$$

The demand faced by each producer $\mathcal{X}_{i,j}^N$ is defined as follows:

$$\mathcal{X}_{i,j}^N = \left(\frac{\mathcal{P}_{i,j}}{\mathcal{P}_j}\right)^{-\epsilon} \frac{\mathcal{Q}_j^N}{N}, \tag{257}$$

[42]By having the term $N^{-\frac{1}{\epsilon-1}}$ in (251), the analysis abstracts from the variety effect and concentrates solely on the effects of markup variation.

and the price index of sector j is given by:

$$\mathcal{P}_j = N^{-\frac{1}{1-\epsilon}} \left(\int_0^N \mathcal{P}_{i,j}^{1-\epsilon} \mathrm{d}i \right)^{\frac{1}{1-\epsilon}}. \tag{258}$$

Combining (253) and (257), the demand for variety $\mathcal{X}_{i,j}^N$ can be expressed in terms of the relative price of the final non-traded good:

$$\mathcal{X}_{i,j}^N = \left(\frac{\mathcal{P}_{i,j}}{\mathcal{P}_j} \right)^{-\epsilon} \left(\frac{\mathcal{P}_j}{P} \right)^{-\omega} \frac{Y^N}{N}. \tag{259}$$

In order to operate, each intermediate good producer must pay a fixed cost denoted by FC measured in terms of the final (non-traded) good which is assumed to be symmetric across firms. Each firm j chooses capital and labor to maximize profits. The profit function for the ith producer in sector j denoted by $\pi_{i,j}^N$ is:

$$\pi_{i,j}^N \equiv \mathcal{P}_j H\left(\mathcal{K}_j^N, \mathcal{L}_j^N\right) - r^K \mathcal{K}_j^N - W \mathcal{L}_j^N - PFC. \tag{260}$$

The demands for capital and hours worked are given by the equalities of the markup-adjusted marginal revenues of capital $\frac{\mathcal{P}_j H_K}{\mu}$ and labor $\frac{\mathcal{P}_j H_L}{\mu}$, to the capital rental rate r^K and the producer wage W, respectively.

K.2 First-Order Conditions

The current-value Hamiltonian for the j-th firm's optimization problem in the non-traded sector is:

$$\mathcal{H}_j^N = \mathcal{P}_j H\left(\mathcal{K}_j^N, \mathcal{L}_j^N\right) - r^K \mathcal{K}_j^N - W \mathcal{L}_j^N - pFC + \eta_j \left[H\left(\mathcal{K}_j^N, \mathcal{L}_j^N\right) - \mathcal{X}_{i,j}^N \right], \tag{261}$$

where $\mathcal{X}_j^N$ stands for the demand for variety j; firm j chooses its price $\mathcal{P}_j$ to maximize profits treating the factor prices as given. First-order conditions for are:

$$\begin{aligned} \mathcal{P}_j H_K + \eta H_K &= r^K, \quad , && (262a) \\ \mathcal{P}_j H_L + \eta H_L &= W, && (262b) \\ \eta_j &= \mathcal{P}_j' H_j, && (262c) \end{aligned}$$

Combining (316a)-(316b) with (316c), by assuming that firms j are symmetric, yields:

$$\mathcal{P}_j H_K \left(1 - \frac{1}{e_j} \right) = r^K, \tag{263a}$$

$$\mathcal{P}_j H_L \left(1 - \frac{1}{e_j} \right) = W, \tag{263b}$$

where we used the fact that $\frac{\mathcal{P}_j'}{\mathcal{P}_j X_{i,j}^N} = -\frac{1}{e_j}$.

We consider a symmetric equilibrium where all firms in the intermediate good sector produce the output level $\mathcal{X}_{i,j}^N = \mathcal{X}^N$ with the same quantities of labor $\mathcal{L}_{i,j}^N = \mathcal{L}^N$ and capital $\mathcal{K}_{i,j}^N = \mathcal{K}^N$. Hence, the aggregate stock of physical capital and hours worked are $K^N = N\mathcal{K}^N$ and $L^N = N\mathcal{L}^N$, respectively. They also set the same price $\mathcal{P}_{i,j} = \mathcal{P}$. Hence, eqs. (254) and (258) imply that $\mathcal{P} = P$.

Remembering that the markup is given by $\mu = \frac{e}{e-1}$, first-order conditions can be rewritten as follows:

$$P \frac{H_K}{\mu} = r^K, \tag{264a}$$

$$P \frac{H_L}{\mu} = W. \tag{264b}$$

We follow Yang and Heijdra [1993] and Jaimovich and Floetotto [2008] by taking into account the influence of the individual price on the sectoral price index:

$$e(N) = \epsilon - \frac{(\epsilon - \omega)}{N}, \quad N \in (1, \infty). \tag{265}$$

As will be useful later, we calculate the partial derivatives of the price-elasticity of demand and the markup with respect to the number of firms:

$$e_N = \frac{\partial e}{\partial N} = \frac{\epsilon - \omega}{N^2} > 0, \quad \mu_N = \frac{\partial \mu}{\partial N} = -\frac{e_N}{(e-1)^2} = -\frac{e_N}{e-1}\frac{\mu}{e} < 0, \tag{266}$$

where we let $\mu = \frac{e}{e-1}$.

We further assume that free entry drives profits down to zero in all industries of the non-traded sector at each instant of time. Using constant returns to scale in production, i. e. $X = H(K, L) = H_K K + H_L L$, and the zero profit condition, in the aggregate, we have:

$$PH\left(K^N, L^N\right) - r^K K^N - WL^N - PNFC = 0. \tag{267}$$

Substituting the short-run static solution for non-traded output (36), the zero-profit condition (267) can be rewritten as:

$$Y^N\left(K, P, \bar{\lambda}, \mu(N)\right)\left(1 - \frac{1}{\mu(N)}\right) = NFC. \tag{268}$$

K.3 Short-Run Static Solution for the Number of Firms

The zero profit condition can be solved for the number of producers in the non-traded sector:

$$N = N\left(K, P, \bar{\lambda}\right), \tag{269}$$

where partial derivatives are given by:

$$N_x \equiv \frac{\partial N}{\partial x} = -\frac{Y_x^N \omega_{FC}}{\chi} \gtrless 0, \tag{270}$$

where $x = K, P, \bar{\lambda}$, $\omega_{FC} \equiv NFC/Y^N$ corresponds to the share of fixed costs in markup-adjusted output and we set

$$\chi = \frac{Y^N}{N}\left\{\left[\eta_{Y^N,\mu}(\mu - 1) + 1\right]\frac{\eta_{\mu,N}}{\mu} - \omega_{FC}\right\}. \tag{271}$$

Inspection of (271) shows that $\chi < 0$ if $\eta_{\mu,N}$ is not too large. This implies that an input inflow in the non-traded sector that raises Y^N and thereby yields to profit opportunities results in firm entry which lowers the markup.

K.4 Equilibrium Dynamics and Formal Solutions

Inserting short-run static solutions for non-traded output and consumption, given by (36) and (28) respectively, into the non-traded good market-clearing condition (12), and inserting short-run static solution for capital-labor ratio in the non-traded good sector (30) into the dynamic equation for the real exchange rate (5d), and substituting the short-run static solution for the number of firms (269) yields:

$$\dot{K} = \frac{Y^N\{K, P, \mu[N(K,P)]\}}{\mu[N(K,P)]} - C^N(P) - \delta_K K - G^N, \tag{272a}$$

$$\dot{P} = P\left\{r^\star + \delta_K - \frac{h_k\left(k^N\{P, \mu[N(K,P)]\}\right)}{\mu[N(K,P)]}\right\}. \tag{272b}$$

For clarity purpose, we dropped variables which are constant over time from short-run static solutions.

Linearizing these two equations around the steady-state, and denoting by $\tilde{x} = \tilde{K}, \tilde{P}$ the steady-state values of $x = K, P$, we obtain in a matrix form:

$$\left(\dot{K}, \dot{P}\right)^T = J\left(K(t) - \tilde{K}, P(t) - \tilde{P}\right)^T, \tag{273}$$

where J is given by

$$J \equiv \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \tag{274}$$

where elements evaluated at the steady-state are:

$$\begin{aligned}
b_{11} &= \frac{Y^N}{\mu}\left[\frac{Y_K^N}{Y^N} - \frac{\mu_N}{\mu}N_K\left(1 - \frac{Y_\mu^N \mu}{Y^N}\right)\right] - \delta_K, && (275a)\\
b_{12} &= \frac{Y^N}{\mu}\left[\frac{Y_P^N}{Y^N} - \frac{\mu_N}{\mu}N_P\left(1 - \frac{Y_\mu^N \mu}{Y^N}\right)\right] - C_P^N, && (275b)\\
b_{21} &= \frac{P}{\mu}h_{kk}\frac{\mu_N N_K}{\mu}k^N\left(\frac{h_k}{h_{kk}k^N} - \frac{k_\mu^N \mu}{k^N}\right), && (275c)\\
b_{22} &= -\frac{P}{\mu}h_{kk}\left[k_P^N - \frac{\mu_N N_P}{\mu}k^N\left(\frac{h_k}{h_{kk}k^N} - \frac{k_\mu^N \mu}{k^N}\right)\right]. && (275d)
\end{aligned}$$

Equilibrium Dynamics

The sign of the determinant denoted by Det of the 2×2 Jacobian matrix (274) is ambiguous:

$$\begin{aligned}
\text{Det J} &= b_{11}b_{22} - b_{12}b_{21} \\
&= \left(\frac{Y_K^N}{\mu} - \delta_K\right)\left[\frac{Y_K^T}{\tilde{P}} + \frac{P}{\mu}h_{kk}k^N\frac{\mu_N N_p}{\mu}\left(\frac{h_k}{h_{kk}k^N} - \frac{k_\mu^N \mu}{k^N}\right)\right] \\
&\quad -\frac{\mu_N}{\mu}N_K\left[\frac{Y^N}{\mu}\left(1 - \frac{Y_\mu^N \mu}{Y^N}\right)\frac{Y_K^T}{\tilde{P}} + \left(\frac{Y_P^N}{\mu} - C_P^N\right)\frac{P}{\mu}h_{kk}k^N\left(\frac{h_k}{h_{kk}k^N} - \frac{k_\mu^N \mu}{k^N}\right)\right] \quad (276)
\end{aligned}$$

and the trace denoted by Tr is given by:

$$\begin{aligned}
\text{Tr J} &= b_{11} + b_{22} = \frac{Y_K^T}{\mu} + \frac{Y_K^N}{P} - \delta_K \\
&\quad -\frac{\mu_N}{\mu}\left[N_K\frac{Y^N}{\mu}\left(1 - \frac{Y_\mu^N \mu}{Y^N}\right) - N_P\frac{P}{\mu}h_{kk}k^N\left(\frac{h_k}{h_{kk}k^N} - \frac{k_\mu^N \mu}{k^N}\right)\right], \\
&= r^\star - \frac{\mu_N}{\mu}N_K\frac{Y^N}{\mu} > 0, && (277)
\end{aligned}$$

where we used the fact that $\frac{Y_K^T}{\mu} + \frac{Y_K^N}{P} = \frac{h_k}{\mu} = r^\star + \delta_K$; the positive sign follows from $N_K > 0$ and $\mu_N < 0$. If the elasticity of the markup to the flow of entry is not too large, then determinant (276) is negative so that the condition for saddle-path stability with real-valued roots holds. Such a condition requires that the markup must be initially not too large.

Characteristic roots from J write as follows:

$$\nu_i \equiv \frac{1}{2}\left\{\text{Tr J} \pm \sqrt{(\text{Tr J})^2 - 4\text{Det J}}\right\} \gtrless 0, \quad i = 1, 2. \tag{278}$$

We denote by $\nu_1 < 0$ and $\nu_2 > 0$ the stable and unstable real eigenvalues, satisfying

$$\nu_1 < 0 < r^\star < \nu_2. \tag{279}$$

Since the system features one state variable, K, and one jump variable, P, the equilibrium yields a unique one-dimensional stable saddle-path.

General solutions are those described by (52) with eigenvector ω_2^i associated with eigenvalue μ_i given by:

$$\omega_2^i = \frac{\nu_i - b_{11}}{b_{12}}. \tag{280}$$

K.5 Formal Solution for the Stock of Foreign Assets

Inserting first short-run static solutions for Y^T and C^T given by (36) and (28), respectively, substituting the short-run static solution for the number of firms given by (269), into eq.(3), and linearizing around the steady-state gives:

$$\dot{B}(t) = r^\star \left(B(t) - \tilde{B}\right) + \left[Y_K^T + Y_\mu^T \mu_N N_K\right] \left(K(t) - \tilde{K}\right) + \left[\left(Y_P^T + Y_\mu^T \mu_N N_P\right) - C_P^T\right] \left(P(t) - \tilde{P}\right), \tag{281}$$

where C_P^T is given by (29b).

Using the fact that $P(t) - \tilde{P} = \omega_2^1 \left(K(t) - \tilde{K}\right)$, setting

$$N_1 = \left[Y_K^T + Y_\mu^T \mu_N N_K\right] + \left[\left(Y_P^T + Y_\mu^T \mu_N N_P\right) - C_P^T\right] \omega_2^1, \tag{282}$$

solving for the differential equation and invoking the transversality condition for intertemporal solvency, the stable solution for net foreign assets finally reduces to:

$$B(t) - \tilde{B} = \Phi_1 \left(K(t) - \tilde{K}\right), \tag{283}$$

and the linearized version of the nation's intertemporal budget constraint is given by:

$$\tilde{B} - B_0 = \Phi_1 \left(\tilde{K} - K_0\right), \tag{284}$$

where we used the fact that $B_1 \equiv K_0 - \tilde{K}$.

K.6 Solutions for L, N, and W

Linearizing the short-run static solution $N = N(K, P)$ yields the solution for the number of firms:

$$\begin{aligned} N(t) &= \tilde{N} + N_K \left(K(t) - \tilde{K}\right) + N_P \left(P(t) - \tilde{P}\right), \\ &= \tilde{N} + \left(N_K + N_P \omega_2^1\right) B_1 e^{\nu_1 t} + \left(N_K + N_P \omega_2^2\right) B_2 e^{\nu_2 t}. \end{aligned} \tag{285}$$

Linearizing the short-run static solution for labor $L = L(P, \mu)$, using the fact that $\mu = \mu(N)$, and substituting the appropriate solutions, the solution for $L(t)$ reads:

$$L(t) = \tilde{L} + L_P \left(P(t) - \tilde{P}\right) + L_\mu \left(\mu(t) - \tilde{\mu}\right), \tag{286}$$

$$= \tilde{L} + L_P \left[\omega_2^1 - \frac{\tilde{P}}{\tilde{\mu}} \mu_N \left(N_K + N_P \omega_2^1\right)\right] B_1 e^{\nu_1 t} + L_P \left[\omega_2^2 - \frac{\tilde{P}}{\tilde{\mu}} \mu_N \left(N_K + N_P \omega_2^2\right)\right] B_1 e^{\nu_2 t} \tag{287}$$

where we used the fact that $L_\mu = -\frac{L_P P}{\mu}$.

Linearizing the short-run static solution for the wage rate $W = W(P,\mu)$ and substituting appropriate solutions yields:

$$\begin{aligned}
W(t) &= \tilde{W} + W_P \omega_2^1 \left(K(t) - \tilde{K}\right) + W_\mu \mu_N \left(N(t) - \tilde{N}\right), \\
&= \tilde{W} + W_P \left[\omega_2^1 - \frac{\tilde{P}}{\tilde{\mu}} \mu_N \left(N_K + N_P \omega_2^1\right)\right] B_1 e^{\nu_1 t} + W_P \left[\omega_2^2 - \frac{\tilde{P}}{\tilde{\mu}} \mu_N \left(N_K + N_P \omega_2^2\right)\right] B_2 e^{\nu_2 t}, \quad (288)
\end{aligned}$$

where we used the fact that $W_\mu = -\frac{W_P P}{\mu}$.

K.7 The Two-Step Procedure: Wealth Effect and Government Spending

By analytical convenience, we rewrite the system of steady-state equations, assuming that $\delta_K = 0$:

$$\frac{h_k\left[k^N\left(\tilde{P},\tilde{\mu}\right)\right]}{\tilde{\mu}} = r^\star, \tag{289a}$$

$$\frac{1}{\tilde{\mu}} Y^N\left(\tilde{K},\tilde{P},\bar{\lambda}\right) - C^N\left(\bar{\lambda},\tilde{P}\right) - G^N = 0, \tag{289b}$$

$$r^\star \tilde{B} + Y^T\left(\tilde{K},\tilde{P},\bar{\lambda}\right) - C^T\left(\bar{\lambda},\tilde{P}\right) - G^T = 0, \tag{289c}$$

together with the intertemporal solvency condition

$$\left(\tilde{B} - B_0\right) = \Phi_1\left(\tilde{K} - K_0\right), \tag{289d}$$

where K_0 and B_0 correspond to the initially predetermined stocks of physical capital and foreign assets, and $\tilde{\mu} = \mu\left(\tilde{K},\tilde{P},\bar{\lambda}\right)$.

Derivation of Steady-State Functions

In a **first step**, we solve the system (289a)-(289c) for $\tilde{P}$, $\tilde{K}$ and $\tilde{B}$ as functions of the marginal utility of wealth, $\bar{\lambda}$, government spending G^N together with the mark-up. Totally differentiating equations (289a)-(289c) yields in matrix form:

$$\begin{pmatrix}
\frac{h_{kk}k_P^N}{\mu} & 0 & 0 \\
\left(\frac{Y_P^N}{\mu} - C_P^N\right) & \left(\frac{Y_K^N}{\mu} - \delta_K\right) & 0 \\
\left(Y_P^T - C_P^T\right) & Y_K^T & r^\star
\end{pmatrix}
\begin{pmatrix}
\mathrm{d}\tilde{P} \\ \mathrm{d}\tilde{K} \\ \mathrm{d}\tilde{B}
\end{pmatrix}$$

$$= \begin{pmatrix}
\frac{Y_K^N}{\mu}\mathrm{d}\mu \\
-\left(\frac{Y_{\bar{\lambda}}^N}{\mu} - C_{\bar{\lambda}}^N\right)\mathrm{d}\bar{\lambda} + \frac{Y^N}{\mu^2}\left(1 + \frac{PY_P^N}{Y^N} + 1\right)\mathrm{d}\mu + \mathrm{d}G^N \\
-\left(Y_{\bar{\lambda}}^T - C_{\bar{\lambda}}^T\right)\mathrm{d}\bar{\lambda} - Y_\mu^T \mathrm{d}\mu
\end{pmatrix}, \tag{290}$$

where we used the fact that $\mu f = P\left[h - h_k\left(k^N - k^T\right)\right]$ and $\frac{h_k}{\mu} = r^\star$ at the steady-state to rewrite $r^\star - h_{kk}k_\mu^N$ as $\frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}^T\right)} = \frac{Y_K^N}{\mu}$, and $-\left(\frac{Y_\mu^N}{\mu} - \frac{Y^N}{\mu^2}\right) = \frac{Y^N}{\mu^2}\left(1 + \frac{PY_P^N}{Y^N} + 1\right)$.

The equilibrium value of the marginal utility of wealth $\bar{\lambda}$, government spending G^N and the markup μ determine the following steady-state values:

$$\begin{aligned}
\tilde{P} &= P(\mu), & (291a) \\
\tilde{K} &= K\left(\bar{\lambda}, G^N, \mu\right), & (291b) \\
\tilde{B} &= B\left(\bar{\lambda}, G^N, \mu\right), & (291c)
\end{aligned}$$

where partial derivatives are given by (217) and (218), and partial derivatives with respect to the markup are:

$$P_\mu \equiv \frac{\partial \tilde{P}}{\partial \mu} = \frac{\frac{Y_K^N}{\mu}}{h_{kk}k_P^N} = -\left(\frac{\tilde{P}}{\mu}\right)^2 \frac{Y_K^N}{Y_K^T} = P^2\frac{h}{f}, \tag{292a}$$

$$K_\mu \equiv \frac{\partial \tilde{K}}{\partial \mu} = \frac{\frac{Y^N}{\mu^2}}{\left(\frac{Y_K^N}{\mu} - \delta_K\right)} - \left\{\frac{\left(r^\star + \delta_K\right)\frac{Y_P^N}{\mu} - \frac{Y_K^N}{\mu}C_P^N}{\frac{Y_K^T}{P^2}\mu\left(\frac{Y_K^N}{\mu} - \delta_K\right)}\right\}, \tag{292b}$$

where we used the fact that $Ph_{kk}k_P^N - Y_K^N = -h_k = -\mu\left(r^\star + \delta_K\right)$ and $h_{kk}k_P^N = -\frac{Y_K^T}{P^2}\mu$.

In order to get more interpretable analytical expressions, we use the negative and positive roots found with a fixed markup since they are good approximates of the negative and positive roots found with an endogenous markup. Partial derivatives (292) can be rewritten as follows:

$$P_\mu = -\frac{\tilde{P}\nu_1}{\mu\nu_2} > 0, \quad \text{if} \quad k^T > k^N, \tag{293a}$$

$$= -\frac{\tilde{P}\nu_2}{\mu\nu_1} > 0, \quad \text{if} \quad k^N > k^T, \tag{293b}$$

$$K_\mu = \frac{Y^N}{\mu^2\nu_1} - \frac{\tilde{P}}{\nu_1\nu_2}\left[\left(r^\star + \delta_K\right)\frac{Y_P^N}{\mu} - \left(\nu_1 + \delta_K\right)C_P^N\right] \lesseqgtr 0, \quad \text{if} \quad k^T > k^N, \tag{293c}$$

$$= \frac{Y^N}{\mu^2\nu_2} - \frac{\tilde{P}}{\nu_1\nu_2}\left[\left(r^\star + \delta_K\right)\frac{Y_P^N}{\mu} - \left(\nu_2 + \delta_K\right)C_P^N\right] > 0, \quad \text{if} \quad k^N > k^T, \tag{293d}$$

where we used the fact that $h_{kk}k_P^N = -\frac{\mu}{P}\frac{Y_K^T}{P}$ to derive the first equality of (293a). In addition, we made use of the following property $Y_\mu^N = -\frac{P}{\mu}Y_P^N$ and $Y_\mu^T = -\frac{P}{\mu}Y_P^T$ to determine (293c)-(293d). Finally, use has been made of property (338) to rewrite $Y_P^T - C_P^T$ and property (39b) to simplify $\mu Y_K^T + \mu Y_K^N$ which is equal to $\tilde{P}\mu r^\star$ in the long-run.

Since the change in the markup modifies the long-run levels of real consumption and labor supply through the steady-state change in the relative price of non tradables, it is convenient to rewrite their steady-state functions, i.e., their short-run static solutions (26) that hold in the long-run, in terms of $\bar{\lambda}$ and μ:

$$C = m\left(\bar{\lambda}, \mu\right), \quad L = n\left(\bar{\lambda}, \mu\right), \tag{294}$$

where partial derivatives are given by (27) evaluated at the steady-state (that's why we substitute respectively the notations m and n for C and L) and

$$m_\mu \equiv \frac{\partial \tilde{C}}{\partial \mu} = \alpha_C\sigma_C\tilde{C}\frac{\nu_1}{\nu_2} < 0, \quad \text{if} \quad k^T > k^N, \tag{295a}$$

$$= \alpha_C\sigma_C\tilde{C}\frac{\nu_2}{\nu_1} < 0, \quad \text{if} \quad k^N > k^T, \tag{295b}$$

$$n_\mu \equiv \frac{\partial \tilde{L}}{\partial \mu} = -\frac{\sigma_L\tilde{L}\tilde{k}^T}{\tilde{W}}\frac{\tilde{P}\tilde{h}}{\tilde{f}}\frac{\tilde{P}r^\star}{\mu^2} < 0. \tag{295c}$$

We computed (295c) as follows: $n_\mu = \frac{\sigma_L\tilde{L}\tilde{k}^T}{\tilde{W}}\frac{\tilde{P}Y_K^N}{\mu Y_K^T}\frac{\tilde{P}r^\star}{\mu}$.

Following the same procedure, i. e. substituting the steady-state function for the real exchange rate into the static solution for wage evaluated at the steady-state, the steady-state function for wage can be rewritten as follows:

$$W = W\left(\mu\right), \tag{296}$$

where the partial derivative w. r. t. μ is given by:

$$W_{\mu} \equiv \frac{\partial \tilde{W}}{\partial \mu} = -\tilde{k}^{T} \frac{\tilde{P}\tilde{h}}{\tilde{f}} \frac{\tilde{P}r^{\star}}{\mu^{2}} < 0, \tag{297}$$

where $W_{\mu} = \tilde{k}^{T} \frac{\tilde{P}Y_{K}^{N}}{\mu Y_{K}^{T}} \frac{\tilde{P}r^{\star}}{\mu}$ with $\frac{Y_{K}^{N}}{Y_{K}^{T}} = -\frac{\tilde{h}}{\tilde{f}} < 0$.

Finally, following a similar procedure, we may express the rental rate of physical capital as a function of μ:

$$r^{K} = r^{K}(\mu), \tag{298}$$

where the partial derivative w. r. t. μ is given by:

$$r_{\mu}^{K} \equiv \frac{\partial \tilde{r}^{K}}{\partial \mu} = -r^{\star} \frac{\tilde{P}}{\mu} \frac{\nu_{1}}{\nu_{2}} > 0, \quad \text{if} \quad k^{T} > k^{N}, \tag{299}$$

$$r_{\mu}^{K} \equiv \frac{\partial \tilde{r}^{K}}{\partial \mu} = -r^{\star} \frac{\tilde{P}}{\mu} \frac{\nu_{2}}{\nu_{1}} > 0, \quad \text{if} \quad k^{N} > k^{T}. \tag{300}$$

Derivation of the Equilibrium Value of the Marginal Utility of Wealth

In a **second step**, we determine the equilibrium change of $\bar{\lambda}$ by taking the total differential of the intertemporal solvency condition (289d):

$$\left[v_{\bar{\lambda}} - \Phi_{1} K_{\lambda}\right] \mathrm{d}\bar{\lambda} = -\left[v_{G^{N}} - \Phi_{1} K_{G^{N}}\right] \mathrm{d}G^{N}, \tag{301}$$

from which may solve for the equilibrium value of $\bar{\lambda}$ as a function of government spending on the non-traded good:

$$\bar{\lambda} = \lambda\left(G^{N}\right), \tag{302}$$

with

$$\lambda_{G^{N}} \equiv \frac{\partial \bar{\lambda}}{\partial G^{N}} = -\frac{\left[v_{G^{N}} - \Phi_{1} K_{G^{N}}\right]}{\left[v_{\bar{\lambda}} - \Phi_{1} K_{\bar{\lambda}}\right]}. \tag{303}$$

L No-Entry

In this section, we develop an alternative version of the two-sector model with a perfectly competitive sector producing a traded good and an imperfectly competitive sector producing a non-traded good. We assume that each producer j of a unique variety of the non-traded good has the following technology $X_{j}^{N} = H\left(\mathcal{K}_{j}, \mathcal{L}_{j}\right)$ with $\mathcal{K}_{j}$ the capital stock and $\mathcal{L}_{j}$ labor. While in section L we consider a model with free entry and endogenous markups, in this section, we solve the model by considering no-entry which implies that the markups are fixed but profits are no longer driven down to zero.

L.1 Framework

The final non-traded output, Y^{N}, is produced in a competitive retail sector using a constant-returns-to-scale production function which aggregates a continuum measure one of sectoral non-traded goods:

$$Y^{N} = \left[\int_{0}^{1} \left(\mathcal{Q}_{j}^{N}\right)^{\frac{\omega-1}{\omega}} \mathrm{d}j\right]^{\frac{\omega}{\omega-1}}, \tag{304}$$

where $\omega > 0$ represents the elasticity of substitution between any two different sectoral goods and $\mathcal{Q}_{j}^{N}$ stands for intermediate consumption of sector'j variety (with $j \in [0, N]$). The final good producers behave competitively, and the households use the final good for both consumption and investment.

In each of the j sectors, there are $N > 1$ firms producing differentiated goods that are aggregated into a sectoral non-traded good by a CES aggregating function. The non-traded output sectoral good j is:[43]

$$\mathcal{Q}_j^N = N^{-\frac{1}{\epsilon-1}} \left[\int_0^N \left(\mathcal{X}_{i,j}^N \right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{305}$$

where $\mathcal{X}_{i,j}^N$ stands for output of firm i in sector j and ϵ is the elasticity of substitution between any two varieties.

Denoting by P and $\mathcal{P}_j$ the relative price of the final good and of the jth variety of the intermediate good, respectively, the profit the final good producer is written as follows:

$$\Pi^N = P \left[\int_0^N \left(\mathcal{Q}_j^N \right)^{\frac{\omega-1}{\omega}} \mathrm{d}j \right]^{\frac{\omega}{\omega-1}} - \int_0^1 \mathcal{P}_j \mathcal{Q}_j^N \mathrm{d}j. \tag{306}$$

Total cost minimizing for a given level of final output gives the (intratemporal) demand function for each input:

$$\mathcal{Q}_j^N = \left(\frac{\mathcal{P}_j}{P} \right)^{-\omega} Y^N, \tag{307}$$

and the price of the final output is given by:

$$P = \left(\int_0^1 \mathcal{P}_j^{1-\omega} \mathrm{d}j \right)^{\frac{1}{1-\omega}}, \tag{308}$$

where $\mathcal{P}_j$ is the price index of sector j and P is the price of the final good.

Within each sector, there is monopolistic competition; each firm that produces one variety $\mathcal{X}_{i,j}^N$ is a price setter. Intermediate output $\mathcal{X}_{i,j}^N$ is produced using capital $\mathcal{K}_{i,j}^N$ and labor $\mathcal{L}_{i,j}^N$:

$$\mathcal{X}_{i,j}^N = H \left(\mathcal{K}_{i,j}^N, \mathcal{L}_{i,j}^N \right). \tag{309}$$

Denoting by $\mathcal{P}_{i,j}$ the price of good i in sector j, the profit function for the jth sector good producer denoted by π_j^N is:

$$\pi_j^N \equiv \mathcal{P}_j N^{-\frac{1}{\epsilon-1}} \left(\int_0^N \left(\mathcal{X}_{i,j}^N \right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^N \mathcal{P}_{i,j} \mathcal{X}_{i,j}^N \mathrm{d}i. \tag{310}$$

The demand faced by each producer $\mathcal{X}_{i,j}^N$ is defined as :

$$\mathcal{X}_{i,j}^N = \left(\frac{\mathcal{P}_{i,j}}{\mathcal{P}_j} \right)^{-\epsilon} \frac{\mathcal{Q}_j^N}{N}, \tag{311}$$

and the price index of sector j is given by:

$$\mathcal{P}_j = N^{-\frac{1}{1-\epsilon}} \left(\int_0^N \mathcal{P}_{i,j}^{1-\epsilon} \mathrm{d}i \right)^{\frac{1}{1-\epsilon}}. \tag{312}$$

Combining (307) and (311), the demand for variety $\mathcal{X}_{i,j}^N$ can be expressed in terms of the relative price of the final non-traded good:

$$\mathcal{X}_{i,j}^N = \left(\frac{\mathcal{P}_{i,j}}{\mathcal{P}_j} \right)^{-\epsilon} \left(\frac{\mathcal{P}_j}{P} \right)^{-\omega} \frac{Y^N}{N}. \tag{313}$$

[43] By having the term $N^{-\frac{1}{\epsilon-1}}$ in (305), the analysis abstracts from the variety effect and concentrates solely on the effects of markup variation.

In order to operate, each intermediate good producer must pay a fixed cost denoted by FC measured in terms of the final good which is assumed to be symmetric across firms. Each firm j chooses capital and labor to maximize profits. The profit function for the ith producer in sector j denoted by $\pi^N_{i,j}$ is:

$$\pi^N_{i,j} \equiv \mathcal{P}_j H\left(\mathcal{K}^N_j, \mathcal{L}^N_j\right) - r^K \mathcal{K}^N_j - W\mathcal{L}^N_j - PFC. \tag{314}$$

The demands for capital and hours worked are given by the equalities of the markup-adjusted marginal revenues of capital $\frac{\mathcal{P}_j H_K}{\mu}$ and labor $\frac{\mathcal{P}_j H_L}{\mu}$, to the capital rental rate r^K and the producer wage W, respectively.

L.2 First-Order Conditions

The current-value Hamiltonian for the j-th firm's optimization problem in the non-traded sector writes as follows:

$$\mathcal{H}^N_j = \mathcal{P}_j H\left(\mathcal{K}^N_j, \mathcal{L}^N_j\right) - r^K \mathcal{K}^N_j - W\mathcal{L}^N_j - PFC + \eta_j\left[H\left(\mathcal{K}^N_j, \mathcal{L}^N_j\right) - \mathcal{X}^N_{i,j}\right], \tag{315}$$

where $\mathcal{X}^N_j$ stands for the demand for variety j; firm j chooses $\mathcal{K}^N_j$ and $\mathcal{L}^N_j$ to maximize profits treating the factor prices as given. First-order conditions for the non-traded sector write as follows:

$$\mathcal{P}_j H_K + \eta H_K = r^K, \quad , \tag{316a}$$

$$\mathcal{P}_j H_L + \eta H_L = W, \tag{316b}$$

$$\eta_j = \mathcal{P}'_j H_j, \tag{316c}$$

Combining (316a)-(316b) with (316c), by assuming that firms j are symmetric, yields:

$$\mathcal{P}_j H_K\left(1 - \frac{1}{e_j}\right) = r^K, \tag{317a}$$

$$\mathcal{P}_j H_L\left(1 - \frac{1}{e_j}\right) = W, \tag{317b}$$

where we used the fact that $\frac{\mathcal{P}'_j}{\mathcal{P}_j X^N_{i,j}} = -\frac{1}{e_j}$.

We consider a symmetric equilibrium where all firms in the intermediate good sector produce the output level $\mathcal{X}^N_{i,j} = \mathcal{X}^N$ with the same quantities of labor $\mathcal{L}^N_{i,j} = \mathcal{L}^N$ and capital $\mathcal{K}^N_{i,j} = \mathcal{K}^N$. Hence, the aggregate stock of physical capital and hours worked are $K^N = N\mathcal{K}^N$ and $L^N = N\mathcal{L}^N$, respectively. They also set the same price $\mathcal{P}_{i,j} = \mathcal{P}$. Hence, eqs. (308) and (312) imply that $\mathcal{P} = P$.

Defining the markup as follows $\mu = \frac{e}{e-1}$, first-order conditions are:

$$P\frac{H_K}{\mu} = r^K, \tag{318a}$$

$$P\frac{H_L}{\mu} = W. \tag{318b}$$

We further assume no-entry so that profits can be positive. Aggregating over the number of competitors, aggregate profits can be rewritten as:

$$\Pi^N \equiv N\pi^N = PH\left(K^N, L^N\right) - r^K K^N - WL^N - PNFC. \tag{319}$$

Using constant returns to scale in production, i. e. $Y^N = H_K K_N + H_L L_N$, substituting the short-run static solution for non-traded output (36), using the fact that $PH_K/\mu = r^K$ and $PH_L/\mu = W$, we have:

$$\Pi^N = \Pi^N\left(K, P, \bar{\lambda}\right) = P\left[Y^N\left(K, P, \bar{\lambda}\right)\left(1 - \frac{1}{\mu}\right) - NFC\right], \tag{320}$$

where the partial derivatives of aggregate profits in the non-traded sector with respect to $K, P, \bar{\lambda}$ are given by:

$$\Pi_P^N \equiv \frac{\partial \Pi^N}{\partial P} = \frac{\Pi^N}{P} + P Y_P^N \left(1 - \frac{1}{\mu}\right) > 0, \tag{321a}$$
$$\Pi_K^N \equiv \frac{\partial \Pi^N}{\partial K} = P Y_K^N \left(1 - \frac{1}{\mu}\right) \gtreqless 0, \tag{321b}$$
$$\Pi_{\bar{\lambda}}^N \equiv \frac{\partial \Pi^N}{\partial \bar{\lambda}} = P Y_{\bar{\lambda}}^N \left(1 - \frac{1}{\mu}\right) \lesseqgtr 0. \tag{321c}$$

L.3 Equilibrium Dynamics

Inserting short-run static solutions (26), (28) and (36) into (5d) and (12), we obtain:

$$\dot{K} = \frac{Y^N\left(K, P, \bar{\lambda}\right)}{\mu} + \frac{\Pi^N\left(K, P, \bar{\lambda}\right)}{P} - C^N\left(\bar{\lambda}, P\right) - \delta_K K - G^N, \tag{322a}$$
$$\dot{P} = P \left\{ r^\star + \delta_K - \frac{h_k\left[k^N(P)\right]}{\mu} \right\}, \tag{322b}$$

where we used the fact that $Y^N - NFC = \frac{Y^N}{\mu} + \frac{\Pi^N}{P}$.

Linearizing these two equations around the steady-state, and denoting $\tilde{x} = \tilde{K}, \tilde{P}$ the steady-state values of $x = K, P$, we obtain in a matrix form:

$$\left(\dot{K}, \dot{P}\right)^T = J\left(K(t) - \tilde{K}, P(t) - \tilde{P}\right)^T, \tag{323}$$

where J is given by

$$J \equiv \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \tag{324}$$

where the elements b_{11}, b_{12}, b_{21}, b_{22} are given by:

$$b_{11} = \frac{Y_K^N}{\mu} + Y_K^N\left(1 - \frac{1}{\mu}\right) - \delta_K = \frac{\tilde{h}}{\left(\tilde{k}^N - \tilde{k}_T\right)} - \delta_K \gtreqless 0, \tag{325a}$$
$$b_{12} = \frac{Y_P^N}{\mu} + \frac{\Pi_P^N}{\tilde{P}} - \frac{\tilde{\Pi}^N}{\tilde{P}^2} - C_P^N = \frac{Y_P^N}{\mu} + Y_P^N\left(1 - \frac{1}{\mu}\right) - C_P^N > 0, \tag{325b}$$
$$b_{21} = 0, \quad b_{22} = -\tilde{P}\frac{h_{kk}k_P^N}{\mu} = -\frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^N - \tilde{k}^T\right)} = \frac{Y_K^T}{\tilde{P}} \lesseqgtr 0. \tag{325c}$$

where we used (321a) to determine (325b).

The determinant denoted by Det of the linearized 2×2 matrix (324) is unambiguously negative:

$$\text{Det J} = b_{11}b_{22} = \frac{Y_K^T}{\tilde{P}}\left[\left(\frac{Y_K^N}{\mu} - \delta_K\right) + Y_K^N\left(1 - \frac{1}{\mu}\right)\right] = \frac{Y_K^T}{\tilde{P}}\left(Y_K^N - \delta_K\right) < 0, \tag{326}$$

and the trace denoted by Tr is given by

$$\text{Tr J} = b_{11} + b_{22} = \frac{1}{\tilde{P}}\left(Y_K^T + \frac{\tilde{P}}{\tilde{\mu}}Y_K^N\right) - \delta_K + Y_K^N\left(1 - \frac{1}{\mu}\right) = r^\star + Y_K^N\left(1 - \frac{1}{\mu}\right) > 0, \tag{327}$$

where we used the fact that at the long-run equilibrium $\frac{h_k}{\mu} = r^\star + \delta_K$.

The characteristic root reads as:

$$\nu_i \equiv \frac{1}{2}\left\{\mathrm{TrJ} \pm \sqrt{(\mathrm{TrJ})^2 - 4\mathrm{DetJ}}\right\} \gtrless 0, \quad i = 1, 2. \tag{328}$$

Using (326) and (327), the characteristic root can be rewritten as follows:

$$\nu_i \equiv \frac{1}{2}\left\{\left(Y_K^N - \delta_K\right) + \frac{Y_K^T}{\tilde{P}} \pm \left[\left(Y_K^N - \delta_K\right) - \frac{Y_K^T}{\tilde{P}}\right]\right\} \gtrless 0, \quad i = 1, 2. \tag{329}$$

We denote by $\nu_1 < 0$ and $\nu_2 > 0$ the stable and unstable real-valued eigenvalues, satisfying

$$\nu_1 < 0 < r^\star < \nu_2. \tag{330}$$

Since the system features one state variable, K, and one jump variable, P, the equilibrium yields a unique one-dimensional stable saddle-path.

General solutions paths are given by :

$$\begin{aligned} K(t) - \tilde{K} &= B_1 e^{\nu_1 t} + B_2 e^{\nu_2 t}, && (331a)\\ P(t) - \tilde{P} &= \omega_2^1 B_1 e^{\nu_1 t} + \omega_2^2 B_2 e^{\nu_2 t}, && (331b) \end{aligned}$$

where we normalized ω_1^i to unity. The eigenvector ω_2^i associated with eigenvalue ν_i is given by

$$\omega_2^i = \frac{\nu_i - b_{11}}{b_{12}}, \tag{332}$$

with b_{11} and b_{12} given by (325a) and (325b), respectively.

Case $k^N > k^T$

This assumption reflects the fact that the capital-labor ratio in the non-traded good sector exceeds the capital-labor in the traded sector. From (329), the stable and unstable eigenvalues can be rewritten as follows:

$$\begin{aligned} \nu_1 &= \frac{Y_K^T}{\tilde{P}} = -\frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^N - \tilde{k}^T\right)} < 0, && (333a)\\ \nu_2 &= Y_K^N - \delta_K = \frac{\tilde{h}}{\left(\tilde{k}^N - \tilde{k}^T\right)} - \delta_K > 0. && (333b) \end{aligned}$$

We sign several useful expressions:

$$\begin{aligned} Y_K^N &= (\nu_2 + \delta_K) > 0, && (334a)\\ Y_K^T &= \tilde{P}\nu_1 < 0, && (334b)\\ \frac{\tilde{P} h_{kk} k_P^N}{\mu} &= -\nu_1 > 0, && (334c)\\ Y_{\bar{\lambda}}^N &= -\frac{1}{\bar{\lambda}}\sigma_L \tilde{L}\tilde{k}^T(\nu_2 + \delta_K) < 0, && (334d)\\ Y_{\bar{\lambda}}^T &= -\frac{1}{\bar{\lambda}}\sigma_L \tilde{L}\tilde{P}\tilde{k}^N \nu_1 > 0. && (334e) \end{aligned}$$

We write out eigenvector ω^i associated with eigenvalue ν_i (with $i = 1, 2$), to determine their signs:

$$\omega^1 = \begin{pmatrix} 1 & (+) \\ \frac{\nu_1 - \nu_2}{\left(Y_P^N - C_P^N\right)} & (-) \end{pmatrix}, \quad \omega^2 = \begin{pmatrix} 1 & (+) \\ 0 & \end{pmatrix}. \tag{335}$$

Case $k^T > k^N$

This assumption reflects the fact that the capital-labor ratio in the traded good sector exceeds the capital-labor ratio in the non traded sector. From (329), the stable and unstable eigenvalues can be rewritten as follows:

$$
\begin{aligned}
\nu_1 &= Y_K^N - \delta_K = \frac{\tilde{h}}{\left(\tilde{k}^N - \tilde{k}^T\right)} - \delta_K < 0, && (336a)\\
\nu_2 &= \frac{Y_K^T}{\tilde{P}} = -\frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^N - \tilde{k}^T\right)} > 0. && (336b)
\end{aligned}
$$

We write out eigenvector ω^i associated with eigenvalue ν_i (with $i = 1, 2$), to determine their signs:

$$
\omega^1 = \begin{pmatrix} 1 & (+) \\ 0 & \end{pmatrix}, \quad \omega^2 = \begin{pmatrix} 0 & \\ \frac{\nu_2 - \nu_1}{\left(Y_P^N - C_P^N\right)} & (+) \end{pmatrix}. \qquad (337)
$$

As in the case of free entry and fixed marked, no entry implies that when the real exchange rate remains unaffected after a permanent fiscal shock (since $\omega_2^1 = 0$) when $k^T >^N$.

We can deduce the signs of several useful expressions:

$$
\begin{aligned}
Y_K^N &= (\nu_1 + \delta_K) < 0, && (338a)\\
Y_K^T &= \tilde{P}\nu_2 > 0, && (338b)\\
\frac{\tilde{P} h_{kk} k_P^N}{\mu} &= -\nu_2 < 0, && (338c)\\
Y_{\bar{\lambda}}^N &= -\frac{1}{\bar{\lambda}}\sigma_L \tilde{L}\tilde{k}^T \left(\nu_1 + \delta_K\right) > 0, && (338d)\\
Y_{\bar{\lambda}}^T &= -\frac{1}{\bar{\lambda}}\sigma_L \tilde{L}\tilde{P}\tilde{k}^N \nu_2 < 0. && (338e)
\end{aligned}
$$

L.4 Current Account Dynamics

In this subsection, we derive the current account equation, the stable path for foreign assets and the intertemporal solvency condition. Substituting the definition of lump-sum taxes Z by using (10), and the market clearing condition for non-traded goods (322a) into (3) we get:

$$
\begin{aligned}
\dot{B} &= r^\star B + r^K K(t) + WL + \Pi^N - P_C C - PI - Z,\\
&= r^\star B + \left(r^K K + WL\right) + \Pi^N - P_C C - G^T - PG^N - P\left(Y^N - C^N - G^N - NFC\right).
\end{aligned}
$$

Using the fact that $L^T + L^N = L$, $K^T + K^N = K$, and substituting the expression of aggregate profits in the non-traded sector, i.e., $\Pi^N = PY^N - WL^N - r^K K^N - PNFC$, the dynamic equation for the current account can be rewritten as follows:

$$
\begin{aligned}
\dot{B} &= r^\star B - C^T - G^T + \left[WL^T + r^K K^T\right] + \left[WL^N + r^K K^N\right] + \left[PY^N - WL^N - r^K K^N - PNFC\right]\\
&\quad -P\left[Y^N - NFC\right],\\
&= r^\star B + Y^T - C^T - G^T, && (339)
\end{aligned}
$$

Inserting general solutions for $K(t)$ and $P(t)$, the solution for the stock of international assets is given by follows:

$$
\dot{B}(t) = r^\star\left(B(t) - \tilde{B}\right) + Y_K^T \sum_{i=1}^{2} B_i e^{\nu_i t} + \left[Y_P^T - C_P^T\right] \sum_{i=1}^{2} B_i \omega_2^i e^{\nu_i t}. \qquad (340)
$$

Solving the differential equation leads to the following expression:

$$B(t) - \tilde{B} = \left[\left(B_0 - \tilde{B}\right) - \Phi_1 B_1 - \Phi_2 B_2\right] e^{r^\star t} + \Phi_1 B_1 e^{\nu_1 t} + \Phi_2 B_2 e^{\nu_2 t}, \tag{341}$$

with

$$\Phi_i = \frac{N_i}{\nu_i - r^\star} = \frac{Y_K^T + \left[Y_P^T - C_P^T\right]\omega_2^i}{\nu_i - r^\star}, \quad i = 1, 2. \tag{342}$$

Invoking the transversality condition for intertemporal solvency, the terms in brackets of equation (341) must be zero and we must set $B_2 = 0$. We obtain the linearized version of the nation's intertemporal budget constraint:

$$B_0 - \tilde{B} = \Phi_1 \left(K_0 - \tilde{K}\right). \tag{343}$$

The stable solution for net foreign assets finally reduces to:

$$B(t) - \tilde{B} = \Phi_1 \left(K(t) - \tilde{K}\right). \tag{344}$$

Case $k^N > k^T$

$$\begin{aligned} N_1 &= Y_K^T + \left(Y_P^T - C_P^T\right)\omega_2^1 = \tilde{P}\nu_1 + \left(Y_P^T - C_P^T\right)\omega_2^1 \gtrless 0, && \text{(345a)} \\ N_2 &= Y_K^T + \left(Y_P^T - C_P^T\right)\omega_2^2, && \text{(345b)} \\ &= Y_K^T = \tilde{P}\nu_1 < 0, && \text{(345c)} \end{aligned}$$

where we used the fact that $\omega_2^2 = 0$. Hence we have:

$$\Phi_2 = \frac{N_2}{\nu_2 - r^\star} = \frac{\tilde{P}\nu_1}{\nu_2 - r^\star}. \tag{346}$$

Case $k^T > k^N$

$$\begin{aligned} N_1 &= Y_K^T + \left(Y_P^T - C_P^T\right)\omega_2^1 = \tilde{P}\nu_2 > 0, && \text{(347a)} \\ N_2 &= Y_K^T + \left(Y_P^T - C_P^T\right)\omega_2^2 = \tilde{P}\nu_2 + \left(Y_P^T - C_P^T\right)\omega_2^2 \lesseqgtr 0, && \text{(347b)} \end{aligned}$$

where we used the fact that $\omega_2^1 = 0$. Hence we have:

$$\Phi_1 = \frac{N_1}{\nu_1 - r^\star} = \frac{\tilde{P}\nu_2}{\nu_1 - r^\star}. \tag{348}$$

L.5 Savings Dynamics

The stock of financial wealth is $A \equiv B + PK$. Differentiating with respect to time, substituting the dynamic equations for foreign bonds (3), capital stock (4), and the real exchange rate (322b), i.e., $\dot{A} = \dot{B} + \dot{P}K + P\dot{K}$, the stock of financial wealth evolves as follows:

$$\dot{A} = r^\star A + WL + \Pi^N - P_C C - Z. \tag{349}$$

Substituting short-run static solutions for the real wage, labor supply, aggregate profits, consumption price index, consumption, eq. (349) can be rewritten as follows:

$$\dot{A} = r^\star A + W(P)L\left(\bar{\lambda}, P\right) + \Pi^N\left(K, P, \bar{\lambda}\right) - P_C(P)\, C\left(P, \bar{\lambda}\right) - G^T - PG^N, \tag{350}$$

where we used the fact that $Z = G^T + PG^N$. Linearizing (350) in the neighborhood of the steady-state, we have:

$$\dot{A} = r^\star \left(A(t) - \tilde{A}\right) + \mathcal{M}\left(P(t) - \tilde{P}\right) + \Pi_K^N \left(K(t) - \tilde{K}\right) \tag{351}$$

where

$$\begin{aligned} \mathcal{M} &= W_P \tilde{L} + \tilde{W} L_P + \Pi_P^N - P_C' \tilde{C} - P_C C_P - G^N, \\ &= W_P \tilde{L}\left(1 + \sigma_L\right) + \frac{\tilde{\Pi}^N}{\tilde{P}} + \tilde{P} Y_P^N \left(1 - \frac{1}{\mu}\right) - \tilde{C}^N \left(1 - \sigma_C\right) - G^N. \end{aligned} \tag{352}$$

The general solution for the stock of financial wealth is:

$$\begin{aligned} A(t) &= \tilde{A} + \left[\left(A_0 - \tilde{A}\right) - \frac{\Pi_K^N + \mathcal{M}\omega_2^1}{\nu_1 - r^\star} B_1 - \frac{\Pi_K^N + \mathcal{M}\omega_2^2}{\nu_2 - r^\star} B_2\right] e^{r^\star t} \\ &\quad + \frac{\Pi_K^N + \mathcal{M}\omega_2^1}{\nu_1 - r^\star} B_1 e^{\nu_1 t} + \frac{\Pi_K^N + \mathcal{M}\omega_2^2}{\nu_2 - r^\star} B_2 e^{\nu_2 t}. \end{aligned} \tag{353}$$

Invoking the transversality condition, we obtain the stable solution for financial wealth:

$$A(t) = \tilde{A} + \frac{\Pi_K^N + \mathcal{M}\omega_2^1}{\nu_1 - r^\star} B_1 e^{\nu_1 t}, \tag{354}$$

and the intertemporal solvency condition

$$\tilde{A} - A_0 = \frac{\Pi_K^N + \mathcal{M}\omega_2^1}{\nu_1 - r^\star}\left(\tilde{K} - K_0\right). \tag{355}$$

L.6 Long-Run Effects of Permanent Fiscal Shocks: The Case of No-Entry

In this subsection, we derive the steady-state effects of permanent fiscal shocks by assuming that labor supply is elastically supplied. To keep things simple, we assume that the traded sector is more capital intensive, i.e. $k^T > k^N$.

Substituting first the appropriate short-un static solutions, the steady-state of the economy is obtained by setting $\dot{K}, \dot{P}, \dot{B} = 0$ and is defined by the following set of equations:

$$\frac{h_k\left[k^N\left(\tilde{P}\right)\right]}{\mu} = r^\star + \delta_K, \tag{356a}$$

$$\frac{Y^N\left(\tilde{K}, \tilde{P}, \bar{\lambda}\right)}{\mu} + \frac{\Pi^N\left(\tilde{K}, \tilde{P}, \bar{\lambda}\right)}{\tilde{P}} - C^N\left(\bar{\lambda}, \tilde{P}\right) - \delta_K \tilde{K} - G^N = 0, \tag{356b}$$

$$r^\star \tilde{B} + Y^T\left(\tilde{K}, \tilde{P}, \bar{\lambda}\right) - C^T\left(\bar{\lambda}, \tilde{P}\right) - G^T = 0, \tag{356c}$$

and the intertemporal solvency condition

$$\left(B_0 - \tilde{B}\right) = \Phi\left(K_0 - \tilde{K}\right), \tag{356d}$$

where we used the fact that $\frac{\Pi^N}{P} + Y^N/\mu = Y^N - NFC$. The steady-state equilibrium composed by these four equations jointly determine $\tilde{P}$, $\tilde{K}$, $\tilde{B}$ and $\bar{\lambda}$.

We totally differentiate the system (356d) evaluated at the steady-state which yields in a matrix form:

$$\begin{pmatrix} \frac{h_{kk} k_P^N}{\mu} & 0 & 0 & 0 \\ \left(Y_P^N - C_P^N\right) & Y_K^N - \delta_K & \left(Y_{\bar{\lambda}}^N - C_{\bar{\lambda}}^N\right) & 0 \\ \left(Y_P^T - C_P^T\right) & Y_K^T & \left(Y_{\bar{\lambda}}^T - C_{\bar{\lambda}}^T\right) & r^\star \\ 0 & -\Phi_1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{d}\tilde{P} \\ \mathrm{d}\tilde{K} \\ \mathrm{d}\bar{\lambda} \\ \mathrm{d}\tilde{B} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathrm{d}G^N \\ \mathrm{d}G^T \\ 0 \end{pmatrix}. \tag{357}$$

The determinant denoted by D' of the matrix (356) of coefficients is given by:

$$D' \equiv \frac{h_{kk} k_P^N}{\mu} \left\{ \left(Y_K^N - \delta_K\right) \left(Y_{\bar{\lambda}}^T - C_{\bar{\lambda}}^T\right) - \left(Y_{\bar{\lambda}}^N - C_{\bar{\lambda}}^N\right) \left[Y_K^T + r^\star \Phi_1\right] \right\} \tag{358}$$

Assuming $k^T > k^N$, then the determinant D' reads as:

$$\begin{aligned} D' &= -\frac{\nu_1 \nu_2}{\tilde{P}\bar{\lambda}} \Bigg\{ \left(\sigma_L \tilde{W} \tilde{L} + \sigma_C P_C \tilde{C}\right) \\ &\quad + \frac{\tilde{P}\left(\nu_1 + \delta_K\right)}{r^\star - \nu_1} \left(1 - \frac{1}{\mu}\right) \left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \left(r^\star + \delta_K\right)\right] \Bigg\} > 0. \end{aligned} \tag{359}$$

where we used the fact that $\nu_2 = r^\star - \nu_1 + \left(\nu_1 + \delta_K\right)\left(1 - \frac{1}{\mu}\right)$. Moreover, we computed the following expression:

$$\begin{aligned} k^N \nu_2 + k^T \left(\nu_1 + \delta_K\right) &= -\frac{k^N f}{P\left(k^N - k^T\right)} + k^T \frac{h}{k^N - k^T}, \\ &= -\frac{W}{P} + Y_K^N \left(1 - \frac{1}{\mu}\right), \\ &= -\frac{W}{P} + k^T \left(\nu_1 + \delta_K\right) \left(1 - \frac{1}{\mu}\right). \end{aligned} \tag{360}$$

The steady-state changes following an unanticipated permanent increase in G^N are given by:

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^N} = 0, \tag{361a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = -\frac{1}{D'} \frac{h_{kk} k_P^N}{\mu} \left(Y_K^T + r^\star \Phi_1\right), \tag{361b}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = \frac{1}{D'} \frac{h_{kk} k_P^N}{\mu} \left(Y_{\bar{\lambda}}^T - C_{\bar{\lambda}}^T\right), \tag{361c}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N} = \Phi_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}. \tag{361d}$$

Assuming that $k^T > k^N$, the steady-state changes for K and $\bar{\lambda}$ can be rewritten as follows:

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = -\frac{\tilde{P}\left(\sigma_L \tilde{L} \tilde{k}^N \nu_2 - \sigma_C \frac{\tilde{C}^T}{\tilde{P}}\right)}{\nu_1 \left[\left(\sigma_L \tilde{W} \tilde{L} + \sigma_C P_C \tilde{C}\right) + \tilde{\Gamma}\right]} \lesseqgtr 0, \tag{362a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = \frac{\tilde{P} \nu_2 \bar{\lambda}}{\left(r^\star - \nu_1\right)} \frac{1}{\left[\left(\sigma_L \tilde{W} \tilde{L} + \sigma_C P_C \tilde{C}\right) + \tilde{\Gamma}\right]} > 0, \tag{362b}$$

where $0 < \frac{\nu_2}{(r^\star - \nu_1)} = \frac{\nu_2}{\nu_2 - (\nu_1 + \delta_K)\left(1 - \frac{1}{\mu}\right)} < 1$ and we have set

$$\tilde{\Gamma} = \frac{\tilde{P}\left(\nu_1 + \delta_K\right)}{r^\star - \nu_1} \left(1 - \frac{1}{\mu}\right) \left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \left(r^\star + \delta_K\right)\right] > 0. \tag{363}$$

L.7 Impact Effects of Permanent Fiscal Shocks: The Case of No-Entry

This section estimates the impact effects of a permanent fiscal expansion when the traded sector is more capital intensive than the non-traded sector. The stable adjustment of the economy is described by a saddle-path in (K, P)-space. The capital stock, the real exchange rate, and the stock of traded bonds evolve according to:

$$K(t) = \tilde{K} + B_1 e^{\nu_1 t}, \tag{364a}$$
$$P(t) = \tilde{P} + \omega_2^1 B_1 e^{\nu_1 t}, \tag{364b}$$
$$B(t) = \tilde{B} + \Phi_1 B_1 e^{\nu_1 t}, \tag{364c}$$

where $\omega_2^1 = 0$, $\Phi_1 = -\frac{\tilde{P}\nu_2}{r^\star - \nu_1}$ if $k^T > k^N$ and with

$$B_1 = K_0 - \tilde{K} = -\mathrm{d}\tilde{K},$$

where we made used the fact that K_0 is predetermined.

We derive below the initial reactions of investment and the current account by assuming that the traded sector is more capital intensive than the non traded sector.

$k^T > k^N$

Differentiating (364a) w.r.t. time, evaluating at time $t = 0$, and substituting (79d), the initial response of investment is:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{perm} = -\nu_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = \frac{\tilde{P}\left(\sigma_L \tilde{L}\tilde{k}^N \nu_2 - \sigma_C \frac{\tilde{C}^T}{\tilde{P}}\right)}{\nu_1 \left[\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right) + \tilde{\Gamma}\right]} \gtrless 0. \tag{365}$$

Using the fact that $\Phi_1 = -\frac{\tilde{P}\nu_2}{r^\star - \nu_1}$, the initial reaction of the current account is:

$$\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{perm} = \frac{\tilde{P}\nu_2}{r^\star - \nu_1}\nu_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = -\frac{\tilde{P}\nu_2}{r^\star - \nu_1}\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{perm} \lessgtr 0. \tag{366}$$

L.8 Effect on Aggregate Profits of a Permanent Fiscal Expansion

Since the wealth of households depends now on the present value of profits, the wealth effect triggered by a fiscal expansion is modified compared to the case of free entry. In this subsection, we compute the change in the present discounted value of profits denoted by Π which is defined as follows:

$$\Pi = \int_0^\infty \Pi^N(t) e^{-r^\star t} \mathrm{d}t. \tag{367}$$

Substituting the short-run static solution for Π^N given by eq. (320) and linearizing around the steady-state, we have:

$$\Pi^N(t) = \tilde{\Pi}^N + \left[\Pi_K^N + \Pi_P^N \omega_2^1\right]\left(K(t) - \tilde{K}\right),$$

where we used the fact that $P(t) - \tilde{P} = \omega_2^1\left(K(t) - \tilde{K}\right)$. We set

$$\Upsilon = \Pi_K^N + \Pi_P^N \omega_2^1. \tag{368}$$

Substituting the linearized version of $\Pi^N(t)$ into eq. (367) and solving yields;

$$\Pi = \frac{\tilde{\Pi}^N}{r^\star} + \frac{\Upsilon B_1}{r^\star - \nu_1}. \tag{369}$$

where we substituted the stable solution for $K(t)$ given by eq. (331a).

Differentiating (369) w.r.t. G^N, the change in the present value of profits is given by

$$\begin{aligned}
\frac{\mathrm{d}\Pi}{\mathrm{d}G^N} &= \frac{\Pi^N_{\bar{\lambda}}}{r^\star}\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} - \frac{\nu_1 \Pi^N_K}{r^\star\left(r^\star - \nu_1\right)}\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}, \\
&= \frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\frac{\left(\nu_1 + \delta_K\right)}{\bar{\lambda} r^\star\left(r^\star - \nu_1\right)}\left(1 - \frac{1}{\mu}\right)\left[\sigma_L \tilde{L}\tilde{P}\tilde{k}^T\left(r^\star + \delta_K\right) - \sigma_C \tilde{P}\tilde{C}^N\right] \\
&\quad - \frac{\tilde{P}\left(\nu_1 + \delta_K\right)}{r^\star\left(r^\star - \nu_1\right)}\left(1 - \frac{1}{\mu}\right), \\
&= -\frac{\tilde{P}\left(1 - \frac{1}{\mu}\right)\left(\nu_1 + \delta_K\right)}{r^\star\left(r^\star - \nu_1\right)}\left\{\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\frac{\left[\sigma_L \tilde{L}\tilde{k}^T\left(r^\star + \delta_K\right) - \sigma_C \tilde{C}^N\right]}{\bar{\lambda}} + 1\right\} > 0, \qquad (370)
\end{aligned}$$

where we used the fact that $B_1 = -\mathrm{d}\tilde{K}$, $\omega_2^1 = 0$ which implies that $\Upsilon = \Pi^N_K$, and $\mathrm{d}\tilde{\Pi}^N = \Pi^N_{\bar{\lambda}}\mathrm{d}\bar{\lambda} + \Pi^N_K \mathrm{d}\tilde{K}$ to get the first line, we used the fact that $\mathrm{d}\tilde{K} = K_{\bar{\lambda}}\mathrm{d}\bar{\lambda} + K_{G^N}\mathrm{d}G^N$ (see eq. (373a)), expression of $K_{\bar{\lambda}}$ given by eq. (374a) and the fact that

$$\Pi_{\bar{\lambda}} = -\frac{\sigma_L \tilde{L}\tilde{P}\tilde{k}^T\left(\nu_1 + \delta_K\right)}{\bar{\lambda}}\left(1 - \frac{1}{\mu}\right) > 0, \qquad (371a)$$

$$\Pi^N_K = \tilde{P}\left(\nu_1 + \delta_K\right)\left(1 - \frac{1}{\mu}\right) < 0. \qquad (371b)$$

L.9 The Effects of Temporary Fiscal Shocks: The Case of Elastic Labor Supply

In this section, we derive formal solutions for temporary shocks under no-entry, by assuming elastic labor supply. The derivations of formal solutions are only possible if we assume $k^T > k^N$ since when sectoral capital intensities are reversed, we are not able to derive useful (i.e., interpretable) expressions.

We first solve the system (356a)-(356c) for $\tilde{P}$, $\tilde{K}$ and $\tilde{B}$ as functions of the marginal utility of wealth, $\bar{\lambda}$ and government spending G^N. Totally differentiating equations (356a)-(356c) yields in matrix form:

$$\begin{aligned}
&\begin{pmatrix} \frac{h_{kk}k^N_P}{\mu} & 0 & 0 \\ \left(Y^N_P - C^N_P\right) & \left(Y^N_K - \delta_K\right) & 0 \\ \left(Y^T_P - C^T_P\right) & Y^T_K & r^\star \end{pmatrix}\begin{pmatrix} \mathrm{d}\tilde{P} \\ \mathrm{d}\tilde{K} \\ \mathrm{d}\tilde{B} \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ -\left(Y^N_{\bar{\lambda}} - C^N_{\bar{\lambda}}\right)\mathrm{d}\bar{\lambda} + \mathrm{d}G^N \\ -\left(Y^T_{\bar{\lambda}} - C^T_{\bar{\lambda}}\right)\mathrm{d}\bar{\lambda} \end{pmatrix}. \qquad (372)
\end{aligned}$$

Steady-state values of K and B can be expressed as functions of the shadow value of wealth and government spending G^N:

$$\tilde{K} = K\left(\bar{\lambda}, G^N\right), \qquad (373a)$$

$$\tilde{B} = B\left(\bar{\lambda}, G^N\right), \qquad (373b)$$

with partial derivatives given by:

$$K_{\bar{\lambda}} \equiv \frac{\partial \tilde{K}}{\partial \bar{\lambda}} = -\frac{\left(Y^N_{\bar{\lambda}} - C^N_{\bar{\lambda}}\right)}{\left(Y^N_K - \delta_K\right)}, \qquad (374a)$$

$$B_{\bar{\lambda}} \equiv \frac{\partial \tilde{B}}{\partial \bar{\lambda}} = \frac{Y^T_K\left(Y^N_{\bar{\lambda}} - C^N_{\bar{\lambda}}\right) - \left(Y^N_K - \delta_K\right)\left(Y^T_{\bar{\lambda}} - C^T_{\bar{\lambda}}\right)}{r^\star\left(Y^N_K - \delta_K\right)}. \qquad (374b)$$

We sign expressions when $k^T > k^N$:

$$K_{\bar{\lambda}} \equiv \frac{\partial \tilde{K}}{\partial \bar{\lambda}} = -\frac{1}{\bar{\lambda}}\frac{1}{\nu_1}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_1+\delta_K\right)\right] > 0, \tag{375a}$$

$$B_{\bar{\lambda}} \equiv \frac{\partial \tilde{B}}{\partial \bar{\lambda}} = \frac{\left\{\sigma_C\left(\tilde{P}\tilde{C}^N\nu_2 - \tilde{C}^T\nu_1\right) + \nu_2\tilde{P}\sigma_L\tilde{L}\left[\tilde{k}^N\nu_1 - \tilde{k}^T\left(\nu_1+\delta_K\right)\right]\right\}}{r^\star \nu_1 \bar{\lambda}} < 0, \quad \text{if} \quad k^T > k^N, \tag{375b}$$

and

$$K_{G^N} \equiv \frac{\partial \tilde{K}}{\partial G^N} = \frac{1}{Y_K^N - \delta_K} = \frac{1}{\nu_1} < 0, \tag{376a}$$

$$B_{G^N} \equiv \frac{\partial \tilde{B}}{\partial G^N} = -\frac{Y_K^T}{\left(Y_K^N - \delta_K\right) r^\star} = -\frac{\tilde{P}\nu_2}{r^\star \nu_1} > 0. \tag{376b}$$

To derive solutions for temporary fiscal shocks, we have to solve the following system:

$$B_1 + B_2 = -K_{\bar{\lambda}}\mathrm{d}\bar{\lambda} - K_{G^N}\mathrm{d}G^N, \tag{377a}$$

$$B_1 e^{\nu_1 \mathcal{T}} + B_2 e^{\nu_2 \mathcal{T}} - B_1' e^{\nu_1 \mathcal{T}} = -K_{G^N}\mathrm{d}G^N, \tag{377b}$$

$$\omega_2^1 B_1 e^{\nu_1 \mathcal{T}} + \omega_2^2 B_2 e^{\nu_2 \mathcal{T}} - \omega_2^1 B_1' e^{\nu_1 \mathcal{T}} = 0, \tag{377c}$$

and

$$B_1 \Upsilon_1 + B_2 \Upsilon_2 + B_{\bar{\lambda}}\mathrm{d}\bar{\lambda} = \Omega_1, \tag{378}$$

where we set

$$\Upsilon_1 \equiv \Phi_1, \tag{379a}$$

$$\Upsilon_2 \equiv \Phi_2 + \left(\Phi_1 - \Phi_2\right) e^{(\nu_2 - r^\star)\mathcal{T}}, \tag{379b}$$

$$\Omega_1 \equiv \left[\left(B_{G^N} - \Phi_1 K_{G^N}\right) e^{-r^\star \mathcal{T}} - B_{G^N}\right] \mathrm{d}G^N. \tag{379c}$$

Adopting the same procedure as described in section K.7, we derive formal expressions below for constants B_1, B_2 and B_1' when $k^T > k^N$. We were unable to derive useful formal expressions with the sector reversal of capital intensities.

Case $k^T > k^N$

When considering elastic labor and no entry, the solutions after a rise in G^N are:

$$\frac{B_1}{\mathrm{d}G^N} = -\frac{\left\{\left[\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right) + \Gamma\right] + \frac{\tilde{P}\nu_2}{r^\star - \nu_1}\left(1 - e^{-r^\star T}\right)\left[\sigma_L \tilde{L}\tilde{k}^T\left(\nu_1+\delta_K\right) - \sigma_C \tilde{C}^N\right]\right\}}{\nu_1\left[\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right) + \Gamma\right]} \gtreqless 0, \tag{380a}$$

$$\frac{B_2}{\mathrm{d}G^N} = 0, \tag{380b}$$

$$\frac{B_1'}{\mathrm{d}G^N} = \frac{B_1}{\mathrm{d}G^N} + K_{G^N} e^{-\nu_1 \mathcal{T}}$$
$$= -\frac{1}{\nu_1}\left\{\left(1 - e^{-\nu_1 T}\right) + \frac{\frac{\tilde{P}\nu_2}{r^\star - \nu_1}\left(1 - e^{-r^\star T}\right)\left[\sigma_L \tilde{L}\tilde{k}^T\left(\nu_1+\delta_K\right) - \sigma_C \tilde{C}^N\right]}{\left[\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right) + \Gamma\right]}\right\} \gtreqless 0, \tag{380c}$$

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \lambda_{G^N}\left(1 - e^{-r^\star \mathcal{T}}\right) > 0, \tag{380d}$$

where λ_{G^N} represents the change in the equilibrium value of the shadow value of wealth after a permanent increase in G^N (see eq. (362b)).

General solutions for K and P are:

$$K(t) - \tilde{K} = B_1 e^{\nu_1 t} + B_2 e^{\nu_2 t}, \tag{381a}$$

$$P(t) - \tilde{P} = \omega_2^1 B_1 e^{\nu_1 t} + \omega_2^2 B_2 e^{\nu_2 t}. \tag{381b}$$

Differentiating eq. (381a) w.r.t. time, evaluating at time $t = 0$ and differentiating w.r.t. G^N, we obtain the initial response of investment following a temporary rise in government spending on the non-traded good:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} = \nu_1 \frac{B_1}{\mathrm{d}G^N} + \nu_2 \frac{B_2}{\mathrm{d}G^N}.$$

Substituting (380a) and using the fact that $\frac{B_2}{\mathrm{d}G^N} = 0$, the initial reaction of investment is given by:

$$\begin{aligned}\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} &= \nu_1 \frac{B_1}{\mathrm{d}G^N}, \\ &= -\left\{1 + \frac{\nu_2}{r^\star - \nu_1}\left(1 - e^{-r^\star T}\right)\frac{\left[\sigma_L \tilde{L}\tilde{k}^T \tilde{P}\left(\nu_1 + \delta_K\right) - \sigma_C \tilde{P}\tilde{C}^N\right]}{\left[\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right) + \Gamma\right]}\right\} \lessgtr 0.\end{aligned} \tag{382}$$

The general solution for the stock of foreign assets is given by:

$$B(t) = \tilde{B} + \left[\left(B_0 - \tilde{B}\right) - \Phi_1 B_1 - \Phi_2 B_2\right] e^{r^\star t} + \Phi_1 B_1 e^{\nu_1 t} + \Phi_2 B_2 e^{\nu_2 t}, \tag{383}$$

Differentiating eq. (383) w.r.t. time, evaluating at time $t = 0$ and differentiating w.r.t. G^N, we obtain the initial response of the current account after a temporary rise in G^N:

$$\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} = r^\star \left[-\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \Phi_1 \frac{B_1}{\mathrm{d}G^N} - \Phi_2 \frac{B_2}{\mathrm{d}G^N}\right] + \nu_1 \frac{B_1 \Phi_1}{\mathrm{d}G^N} + \nu_2 \frac{B_2 \Phi_2}{\mathrm{d}G^N}.$$

Using the fact that

$$\begin{aligned}&-\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \Phi_1 \frac{B_1}{\mathrm{d}G^N} - \Phi_2 \frac{B_2}{\mathrm{d}G^N} \\ &= -\left[\left(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right)\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + \left(B_{G^N} - \Phi_1 K_{G^N}\right)\right], \\ &= -\left(B_{G^N} - \Phi_1 K_{G^N}\right) e^{-r^\star T} = -\frac{\tilde{P}\nu_2}{r^\star\left(r^\star - \nu_1\right)} e^{-r^\star T},\end{aligned} \tag{384}$$

the initial reaction of the current account can be rewritten as follows:

$$\begin{aligned}\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} &= -\frac{\tilde{P}\nu_2}{\left(r^\star - \nu_1\right)} e^{-r^\star T} - \nu_1 \frac{\tilde{P}\nu_2}{\left(r^\star - \nu_1\right)} \frac{B_1}{\mathrm{d}G^N}, \\ &= \frac{\tilde{P}\nu_2}{r^\star - \nu_1}\left(1 - e^{-r^\star T}\right)\left[1 + \frac{\nu_2}{r^\star - \nu_1}\frac{\left(\sigma_L \tilde{L}\tilde{k}^T \tilde{P}\left(\nu_1 + \delta_K\right) - \sigma_C \tilde{P}\tilde{C}^N\right)}{\left[\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right) + \Gamma\right]}\right] \gtrless 0, \\ &= -\frac{\tilde{P}\nu_2}{\left(r^\star - \nu_1\right)} \tilde{P} e^{-r^\star T} \\ &+ \frac{\tilde{P}\nu_2}{\left(r^\star - \nu_1\right)}\left\{1 + \frac{\tilde{P}\nu_2}{\left(r^\star - \nu_1\right)}\left(1 - e^{-r^\star T}\right)\frac{\left[\sigma_L \tilde{L}\tilde{k}^T\left(\nu_1 + \delta_K\right) - \sigma_C \tilde{C}^N\right]}{\left[\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right) + \tilde{\Gamma}\right]}\right\} \gtrless 0,\end{aligned} \tag{385}$$

where we used the fact that $\Phi_1 = -\frac{\tilde{P}\nu_2}{r^\star - \nu_1}$.

L.10 Effect on Aggregate Profits of a Temporary Fiscal Expansion

Since the wealth of households depends now on the present value of profits, the wealth effect triggered by a fiscal expansion is modified compared to that under free entry. In this subsection, we compute the change in the present discounted value of profits after a temporary fiscal expansion. Hence, the present value of profits denoted by Π evaluated over two sub-periods $(0, \mathcal{T})$ and $(\mathcal{T}, \infty)$ is:

$$\Pi = \int_0^{\mathcal{T}} \Pi^N(t) e^{-r^\star t} \mathrm{d}t + \int_{\mathcal{T}}^{\infty} \Pi^N(t) e^{-r^\star t} \mathrm{d}t. \tag{386}$$

The linearized versions of aggregate profits in the non-traded sector over period 1 (say over $(0, T)$) and over period 2 (say over (T, ∞)) are:

$$\begin{aligned} \Pi^N(t) &= \tilde{\Pi}_1^N + \Pi_K^N \left(K(t) - \tilde{K}_1 \right) = \tilde{\Pi}_1^N + \Pi_K^N B_1 e^{\nu_1 t}, \\ \Pi^N(t) &= \tilde{\Pi}_2^N + \Pi_K^N \left(K(t) - \tilde{K}_2 \right) = \tilde{\Pi}_2^N + \Pi_K^N B_1' e^{\nu_1 t}, \end{aligned}$$

where we used the fact that $\omega_2^1 = 0$ so that the dynamics for the relative price degenerate and the fact that the constant $B_2 = 0$.

Substituting linearized versions of $\Pi^N(t)$ for periods $(0, T)$ and (T, ∞) into eq. (386) and solving yields:

$$\begin{aligned} \Pi &= \frac{\tilde{\Pi}_1^N \left(1 - e^{-r^\star T}\right)}{r^\star} + \Pi_K^N B_1 \frac{\left(1 - e^{-(r^\star - \nu_1)T}\right)}{(r^\star - \nu_1)} + \frac{\tilde{\Pi}_2^N e^{-r^\star T}}{r^\star} \\ &\quad + \Pi_K^N B_1' \frac{e^{-(r^\star - \nu_1)T}}{(r^\star - \nu_1)}. \end{aligned} \tag{387}$$

Using the fact that $B_1 + B_2 = -K_{\bar{\lambda}} \mathrm{d}\bar{\lambda} - K_{G^N} \mathrm{d}G^N$, with $B_2 = 0$ and $K_{G^N} = 1/\nu_1$, and differentiating eq. (387) w.r.t. G^N, the change in the present value of profits after a temporary fiscal expansion is given by

$$\begin{aligned} \left.\frac{\mathrm{d}\Pi}{\mathrm{d}G^N}\right|_{temp} &= \frac{\Pi_{\bar{\lambda}}^N}{r^\star} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - \frac{\nu_1 \Pi_K^N}{r^\star (r^\star - \nu_1)} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - \frac{\Pi_K^N \left(1 - e^{-r^\star T}\right)}{r^\star (r^\star - \nu_1)} \\ &= \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} \frac{(\nu_1 + \delta_K)}{\bar{\lambda} r^\star (r^\star - \nu_1)} \left(1 - \frac{1}{\mu}\right) \left[\sigma_L \tilde{L} \tilde{P} \tilde{k}^T (r^\star + \delta_K) - \sigma_C \tilde{P} \tilde{C}^N\right] \\ &\quad - \frac{\tilde{P} (\nu_1 + \delta_K)}{r^\star (r^\star - \nu_1)} \left(1 - \frac{1}{\mu}\right), \\ &= -\frac{\tilde{P} \left(1 - \frac{1}{\mu}\right) (\nu_1 + \delta_K)}{r^\star (r^\star - \nu_1)} \left(1 - e^{-r^\star T}\right) \left\{\lambda_{G^N} \left[\sigma_L \tilde{L} \tilde{k}^T (r^\star + \delta_K) - \sigma_C \tilde{C}^N\right] + 1\right\} > 0, \\ &= \left(1 - e^{-r^\star T}\right) \left.\frac{\mathrm{d}\Pi}{\mathrm{d}G^N}\right|_{perm}, \end{aligned} \tag{388}$$

where we used the fact that $\mathrm{d}\tilde{\Pi}_1^N = \tilde{\Pi}_1^N - \tilde{\Pi}_0^N = \Pi_K^N \mathrm{d}\tilde{K}_1 + \Pi_{\bar{\lambda}}^N \mathrm{d}\bar{\lambda}$ and $\mathrm{d}\tilde{\Pi}_2^N = \tilde{\Pi}_2^N - \tilde{\Pi}_0^N = \Pi_K^N \mathrm{d}\tilde{K} + \Pi_{\bar{\lambda}}^N \mathrm{d}\bar{\lambda}$, and $\mathrm{d}\tilde{K}_1 = \tilde{K}_1 - K_0 = K_{\bar{\lambda}} \mathrm{d}\bar{\lambda} + K_{G^N} \mathrm{d}G^N$, and $\mathrm{d}\tilde{K} = \tilde{K}_2 - \tilde{K}_0 = K_{\bar{\lambda}} \mathrm{d}\bar{\lambda}$ and collected terms to get the first line, we factorize by $\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp}$ and substitute expressions of Π_K^N and $\Pi_{\bar{\lambda}}^N$ given by eq. (371) to get the second line, substitute the expression of the change in the equilibrium value of the marginal utility of wealth given by (380d) and (362b) to get the third line. Eq. (370) shows that the change in the present value of profits is a scaled-down version of the change after a permanent fiscal shock.

When $k^N > k^T$, we computed the present value of profits numerically by adopting a similar procedure. First, linearizing versions of aggregate profits in the non-traded sector over period 1 (say over $(0, \mathcal{T})$) and over period 2 (say over $(\mathcal{T}, \infty)$) are:

$$\begin{aligned}
\Pi^N(t) &= \tilde{\Pi}_1^N + \Theta^1 \left(K(t) - \tilde{K}_1\right), \\
&= \tilde{\Pi}_1^N + \Theta^1 B_1 e^{\nu_1 t} + \Theta^2 B_2 e^{\nu_2 t}, \\
\Pi^N(t) &= \tilde{\Pi}_2^N + \Theta^2 \left(K(t) - \tilde{K}_2\right) = \tilde{\Pi}_2^N + \Theta^1 B_1' e^{\nu_1 t},
\end{aligned}$$

where $\Theta^1 = \Pi_K^N + \Pi_P^N \omega_2^1$ since the relative price dynamics do no longer degenerate and $\Theta^2 = \Pi_K^N$ since $\omega_2^2 = 0$.

Substituting linearized versions of $\Pi^N(t)$ for periods $(0, T)$ and (T, ∞) into eq. (386) and solving yields:

$$\begin{aligned}
\Pi &= \frac{\tilde{\Pi}_1^N \left(1 - e^{-r^\star \mathcal{T}}\right)}{r^\star} + \Theta^1 B_1 \frac{\left(1 - e^{-(r^\star - \nu_1)\mathcal{T}}\right)}{(r^\star - \nu_1)} + \Theta^2 B_2 \frac{\left(1 - e^{-(r^\star - \nu_2)\mathcal{T}}\right)}{(r^\star - \nu_2)} \\
&\quad + \frac{\tilde{\Pi}_2^N e^{-r^\star \mathcal{T}}}{r^\star} + \Theta^1 B_1' \frac{e^{-(r^\star - \mu_1)\mathcal{T}}}{(r^\star - \mu_1)}. \qquad (389)
\end{aligned}$$

M Solving a Two-Sector Model with Endogenous Markup

In this section, we provide the main steps to solve the two-sector model without capital accumulation. This allows us to isolate the fiscal policy transmission in open economy with endogenous markups.

The small open economy produces a traded and a non traded good by means of a production technology described by linearly homogenous production functions that use labor only. As previously, the output of the non traded good (Y^N) can be used for private (C^N) and public consumption (G^N). The output of the traded good (Y^T) can be consumed by households (C^T) and the government (G^T), or can be exported ($Y^T - C^T - G^T$).

M.1 First-Order Conditions

Households

Households decide on consumption and worked hours by maximizing lifetime utility (2) subject to the flow budget constraint given by:

$$\dot{B}(t) = r^\star B(t) + W(t)L(t) - Z(t) - P_C\left(P(t)\right)C(t). \qquad (390)$$

Denoting the co-state variable associated with eq. (390) by λ, the first-order conditions characterizing the representative household's optimal plans are:

$$C = \left(P_C \lambda\right)^{-\sigma_C}, \qquad (391a)$$

$$L = \left(\frac{\lambda}{\gamma} W\right)^{\sigma_L}, \qquad (391b)$$

$$\dot{\lambda} = \lambda\left(\beta - r^\star\right), \qquad (391c)$$

and the appropriate transversality conditions. We impose $\beta = r^\star$ in order to generate an interior solution for the marginal utility of wealth. Hence, we have $\lambda = \bar{\lambda}$.

Firms

There are two sectors in the economy: a perfectly competitive sector which produces a traded good denoted by the superscript T and an imperfectly competitive sector which produces a non-traded good denoted by the superscript N. Both the traded and non-traded sectors use labor, L^T and L^N, according to linearly homogenous production functions. The technology of production in the traded sector is described by the following production function: $Y^Y = L^T$. We assume that each producer of a unique variety of the non-traded good has the following technology $X_j^N = \mathcal{L}_j^N$ where $\mathcal{L}_j$ is labor. In this subsection, we emphasize the main changes. Further details about the imperfectly competitive non-traded sector and the derivation of endogenous markups can be found in section K.

Both sectors face a labor cost equal to the wage rate W. Combining first order conditions derived from profit-maximization which state that factors are paid to their respective marginal products together with the assumption of perfect mobility of labor across sectors yields:

$$1 = \frac{P}{\mu} = W. \tag{392}$$

Hence, according to (392), the real exchange rate is equal to the markup while the wage rate must remain unchanged:

$$P = \mu, \quad \text{and} \quad W = 1. \tag{393}$$

We follow Yang and Heijdra [1993] and Jaimovich and Floetotto [2008] by taking into account the influence of the individual price on the sectoral price index:

$$e(N) = \epsilon - \frac{(\epsilon - \omega)}{N}, \quad N \in (1, \infty). \tag{394}$$

Because the price-elasticity of demand e increases with the number of firms N, the markup $\mu = \frac{e}{e-1}$ is decreasing as the number of competitors rise:

$$\mu = \mu(N), \quad \mu_N < 0. \tag{395}$$

The partial derivatives of the price-elasticity of demand and the markup with respect to the number of firms are:

$$\eta_{e,N} = \frac{\partial e}{\partial N}\frac{N}{e} = \frac{\epsilon - \omega}{Ne} > 0, \quad \eta_{\mu,N} = \frac{\partial \mu}{\partial N}\frac{N}{\mu} = -\eta_{e,N}\frac{1}{e-1} < 0. \tag{396}$$

We further assume that free entry drives profits down to zero in all industries of the non-traded sector at each instant of time. Using constant returns to scale in production and denoting by L^N aggregate labor in the non-traded sector with $L_N = N\mathcal{L}_N$, the zero profit condition (in the aggregate) is:

$$PL^N - WL^N - PN\psi = 0. \tag{397}$$

where ψ corresponds to fixed costs. Using (392), the zero-profit condition (397) can be rewritten as:

$$L^N\left(1 - \frac{1}{\mu(N)}\right) = N\psi. \tag{398}$$

Aggregating labor over the two sectors, gives us the resource constraint for labor:

$$L^T + L^N = L, \tag{399}$$

where $L_N = N\mathcal{L}_N$.

M.2 Short-run Static Solutions

Inserting $W = 1$, eq. (391b) can be rewritten as follows:

$$L = \left(\frac{\bar{\lambda}}{\gamma}\right)^{\sigma_L}. \tag{400}$$

Solving (391a) and (391b), we get:

$$C = C\left(\bar{\lambda}, P\right), \quad L = L\left(\bar{\lambda}\right), \tag{401}$$

where partial derivatives are given by:

$$\mathrm{d}C = -\sigma_C \frac{C}{\bar{\lambda}} \mathrm{d}\bar{\lambda} - \sigma_C \alpha_C \frac{C}{P} \mathrm{d}P, \tag{402a}$$

$$\mathrm{d}L = \sigma_L \frac{L}{\bar{\lambda}} \mathrm{d}\bar{\lambda}. \tag{402b}$$

Inserting short-run solution for consumption (401) into intra-temporal allocations between non tradable and tradable goods, allows us to solve for C^T and C^N:

$$C^T = C^T\left(\bar{\lambda}, P\right), \quad C^N = C^N\left(\bar{\lambda}, P\right), \tag{403}$$

where

$$\mathrm{d}C^T = -\sigma_C \frac{C^T}{\bar{\lambda}} \mathrm{d}\bar{\lambda} + \alpha_C \frac{C^T}{P}\left(\phi - \sigma_C\right) \mathrm{d}P, \tag{404a}$$

$$\mathrm{d}C^N = -\sigma_C \frac{C^N}{\bar{\lambda}} \mathrm{d}\bar{\lambda} - \frac{C^N}{P}\left[\left(1 - \alpha_C\right)\phi + \alpha_C \sigma_C\right) \mathrm{d}P, \tag{404b}$$

The zero profit condition (398) can be solved for the number of producers in the non-traded sector:

$$N = N\left(L^N\right), \tag{405}$$

where the number of competitors is positively related with production in the non-traded sector L^N as shown formally below:

$$\frac{\partial N}{\partial L^N} = \frac{N\psi}{L^N\left(\psi - \frac{L^N}{N\mu}\eta_{\mu,N}\right)} > 0, \tag{406}$$

where $\eta_{\mu,N} < 0$ is the elasticity of the markup to the number of competitors and is defined by (396).

Eq. (393) which states that $P = \mu$ can be solved for the relative price of non-tradables by inserting (405):

$$P = P\left(L^N\right). \tag{407}$$

Differentiating implies a negative relationship between the relative price of non-tradables and labor in the non-traded sector:

$$\frac{\partial P}{\partial L^N} = \eta_{\mu,N}\eta_{N,L^N}\frac{P}{L^N} < 0, \tag{408}$$

where $\eta_{N,L^N} = N_{L^N}L^N/N > 0$ (with N_{L^N} given by (406)) is the elasticity of the number of competitors to non-traded output $Y^N = L^N$.

M.3 Market Clearing Conditions

Inserting short-run static solutions for C^N given by (403) into the non-traded good market clearing condition gives us:

$$\frac{L^N}{\mu} = C^N\left(\bar{\lambda}, P\right) + G^N. \tag{409}$$

The non-traded good market clearing condition can be solved for non-traded labor by using the fact that $P = \mu$ and inserting the short-run static solution (407) for the relative price of non-tradables:

$$L^N = L^N\left(\bar{\lambda}, G^N\right), \tag{410}$$

where $L^N_{\bar{\lambda}} < 0$ and $L^N_{G^N} > 0$. The effects of $\bar{\lambda}$ and G^N on L^N can be derived as follows. Insert first (407), totally differentiate (400), denote by $\omega_C = \frac{P_C C}{Y}$ the ratio of consumption expenditure to GDP, and by $\omega_N = \frac{PY^N/\mu}{Y} = \frac{L^N}{L}$ (remembering that $Y = Y^T + \frac{P}{\mu}Y^N = L^T + L^N$ since $P = \mu$, $Y^T = L^T$ and $Y^N = L^N$) the ratio of non-traded output less fixed costs to GDP, we have:

$$\mathrm{d}L^N = -\frac{\omega_C \alpha_C \sigma_C L^N}{\bar{\lambda}\chi\omega_N}\mathrm{d}\bar{\lambda} + \frac{P}{\chi}\mathrm{d}G^N, \tag{411}$$

where we used the fact that $\frac{\mu C^N}{L^N} = \frac{PC^N}{L^N} = \frac{\alpha_C \omega_C}{\omega_N}$ and we set

$$\chi = 1 - \eta_{\mu,N}\eta_{N,L^N}\left\{1 - \frac{\alpha_C\omega_C}{\omega_N}\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right]\right\} > 0. \tag{412}$$

While the sign of (412) is ambiguous, because $\eta_{\mu,N}$ is small, χ is positive.

Combining the short-run static solution for the relative price of non-tradables $P = P\left(L^N\right)$ together with the short-run static solution for non-traded labor (410), we have:

$$P = P\left(\bar{\lambda}, G^N\right), \tag{413}$$

with $P_{\bar{\lambda}} = P_{L^N}L^N_{\bar{\lambda}} > 0$ and $P_{G^N} = P_{L^N}L^N_{G^N} < 0$

Inserting first the short-run static solution for labor supply (401) and for non-traded labor (410), the resource constraint for labor can be solved for L^T, i.e., $L^T = L\left(\bar{\lambda}\right) - L^N\left(\bar{\lambda}, G^N\right)$. We get:

$$L^T = L^T\left(\bar{\lambda}, G^N\right), \tag{414}$$

where $L^T_{\bar{\lambda}} = L_{\bar{\lambda}} - L^N_{\bar{\lambda}} > 0$, $L^T_{G^N} = -L^N_{G^N} < 0$.

M.4 Equilibrium Dynamics and Formal Solutions

Combining the non-traded good market clearing condition (409) and the flow budget constraint yields the current account equation:

$$\dot{B}(t) = r^\star B(t) + L^T - C^T - G^T. \tag{415}$$

Inserting the short-run static solution for traded labor (414) and for consumption in tradables (403) together with (413), into the current account equation (415) yields:

$$\dot{B}(t) = r^\star B(t) + L^T\left(\bar{\lambda}, G^N\right) - C^T\left[\bar{\lambda}, P\left(\bar{\lambda}, G^N\right)\right] - G^T. \tag{416}$$

Keeping in mind that the marginal utility of wealth is constant over time, i.e., $\lambda(t) = \bar{\lambda}$, and linearizing (416), we get:

$$\dot{B}(t) = r^\star\left(B(t) - \tilde{B}\right). \tag{417}$$

The general solution is:

$$B(t) = \tilde{B} + D_2 e^{r^\star t}, \tag{418}$$

where D_2 is a constant to be determined. Invoking the transversality condition, the stable solution is:

$$B(t) = \tilde{B}, \tag{419}$$

and the intertemporal solvency condition (ISC) reads:

$$\tilde{B} = B_0. \tag{420}$$

M.5 Steady-State

Inserting the ISC (420) and appropriate short-run static solutions which obviously hold in the long-run, the steady-state can be reduced to one equation

$$r^\star B_0 + L^T\left(\bar{\lambda}, G^N\right) - C^T\left[\bar{\lambda}, P\left(\bar{\lambda}, G^N\right)\right] - G^T = 0. \tag{421}$$

Equation (421) can be solved for the marginal utility of wealth:

$$\bar{\lambda} = \lambda\left(G^N\right). \tag{422}$$

Totally differentiating (421) yields:

$$\left\{\sigma_L \frac{\tilde{L}}{\bar{\lambda}} + \sigma_C \frac{\tilde{C}^T}{\bar{\lambda}} - L^N_{\bar{\lambda}}\left[1 + \frac{\tilde{C}^T}{\tilde{P}}\alpha_C\left(\phi - \sigma_C\right)\eta_{\mu,N}\eta_{N,L^N}\frac{\tilde{P}}{\tilde{L}^N}\right]\right\}\mathrm{d}\bar{\lambda}$$
$$-L^N_{G^N}\left[1 + \frac{\tilde{C}^T}{\tilde{P}}\alpha_C\left(\phi - \sigma_C\right)\eta_{\mu,N}\eta_{N,L^N}\frac{\tilde{P}}{\tilde{L}^N}\right]\mathrm{d}G^N.$$

Dividing both sides by GDP (i.e., $Y = L$), using the fact that $\frac{C^T}{L^N} = \frac{(1-\alpha_C)\omega_C}{\omega_N}$ and $\frac{C^T}{L} = (1-\alpha_C)\,\omega_C$ where we set $\omega_N = PY^N/\mu/Y = L^N/L$ and $\omega_C = P_C C/Y$, and collecting terms, the change in the marginal utility of wealth is given by:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} = \frac{\tilde{P}\bar{\lambda}}{\tilde{Y}\chi}\frac{\left[1 + \frac{\omega_C(1-\alpha_C)\alpha_C}{\omega_N}\left(\phi - \sigma_C\right)\eta_{\mu,N}\eta_{N,L^N}\right]}{\Psi} > 0, \tag{423}$$

where we set

$$\Psi = \sigma_L + \sigma_C\left(1-\alpha_C\right)\omega_C + \frac{\alpha_C\omega_C\sigma_C}{\chi}\left[1 + \frac{\left(1-\alpha_C\right)\omega_C}{\omega_N}\alpha_C\left(\phi - \sigma_C\right)\eta_{\mu,N}\eta_{N,L^N}\right] > 0. \tag{424}$$

Eq. (424) has a positive sign as long as $\eta_{\mu,N} < 0$ is not too large which holds when μ takes reasonable values. Hence, a rise in government spending raises the marginal utility of wealth $\bar{\lambda}$ and thereby yields a negative wealth effect which exerts a negative impact on consumption and a positive influence on labor supply.

Totally differentiating (410) (using (411)) and substituting (423), the change in non-traded labor is given by:

$$\left.\frac{\mathrm{d}L^N}{\mathrm{d}G^N}\right|_{perm} = L^N_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} + L^N_{G^N} = \frac{P\left[\sigma_L + \omega_C\left(1-\alpha_C\right)\sigma_C\right]}{\Psi\chi} > 0, \tag{425}$$

where $\chi > 0$ is given by (412) and $\Psi > 0$ is given by (424). Because non-traded labor rises, profit opportunities induce firm to enter the market (see eq. (406)); since the number of competitors increases, the markup falls (see eq. (395)); as a result, intermediate-good producers in the non-traded sector set lower prices which results in a real exchange rate depreciation (see eq. (408)).

M.6 Derivation of Steady-State Solutions

In this section, we derive steady-state solutions in the case of an endogenous markup. The steady-state reduces to two equations:

$$r^{\star}\tilde{B} + \tilde{L}^{T} - \tilde{C}^{T} - G^{T} = 0, \tag{426a}$$

together with the intertemporal solvency condition

$$\tilde{B} = B_0, \tag{426b}$$

which jointly solve for the stock of traded bonds $\tilde{B}$ and the marginal utility of wealth $\bar{\lambda}$.

We first solve the system (426a) for $\tilde{B}$ as a function of the marginal utility of wealth, $\bar{\lambda}$ and government spending G^N. To do so, substitute solutions for traded labor (414) and for consumption in tradables (403), into the traded good market clearing condition (426a), by inserting first solution for the relative price of non-tradables (413) $P = P\left(\lambda, G^N\right)$, we have:

$$r^{\star}\tilde{B} + L^{T}\left(\bar{\lambda}, G^N\right) - C^{T}\left[\bar{\lambda}, P\left(\bar{\lambda}, G^N\right)\right] - G^{T} = 0. \tag{427}$$

Solving for (428), we get the steady-state value of B can be expressed as a function of the shadow value of wealth and government spending G^N:

$$\tilde{B} = B\left(\bar{\lambda}, G^N\right), \tag{428}$$

with partial derivatives given by:

$$B_{\bar{\lambda}} \equiv \frac{\partial \tilde{B}}{\partial \bar{\lambda}} = -\frac{\left(L^{T}_{\bar{\lambda}} - C^{T}_{\bar{\lambda}} - C^{T}_{P} P_{\bar{\lambda}}\right)}{r^{\star}}, \tag{429a}$$

$$B_{G^N} \equiv \frac{\partial \tilde{B}}{\partial G^N} = -\frac{\left(L^{T}_{G^N} - C^{T}_{P} P_{G^N}\right)}{r^{\star}}, \tag{429b}$$

where $L^{T}_{\bar{\lambda}} = L_{\bar{\lambda}} - L^{N}_{\bar{\lambda}} > 0$, $P_{\bar{\lambda}} = P_{L^N} L^{N}_{\bar{\lambda}} > 0$, and $L^{T}_{G^N} = -L^{N}_{G^N} < 0$, $P_{G^N} = P_{L^N} L^{N}_{G^N} < 0$. Using (411), partial derivatives (429) reduce to:

$$B_{\bar{\lambda}} = -\frac{Y\Psi}{r^{\star}\bar{\lambda}} < 0, \tag{430a}$$

$$B_{G^N} = \frac{P\left[1 + \frac{\omega_C(1-\alpha_C)\alpha_C}{\omega_N}\left(\phi - \sigma_C\right)\eta_{\mu,N}\eta_{N,L^N}\right]}{r^{\star}\chi} > 0, \tag{430b}$$

where $\Psi > 0$ is given by (424).

M.7 Derivation of Formal Solutions after Temporary Fiscal Shocks

In this section, we provide the main steps to derive formal solutions for key variables after a temporary unanticipated fiscal shock, by applying the procedure developed by Schubert and Turnovsky [2002].

The small open economy is initially in steady-state equilibrium, i.e., $\lambda = \lambda_0$ and $B = B_0$. We suppose now that government expenditure changes unexpectedly at time $t = 0$ from the original level G^N_0 to level G^N_1 over the period $0 \leq t < \mathcal{T}$, and reverts back at time $\mathcal{T}$ permanently to its initial level $G^N_{\mathcal{T}} = G^N_2 = G^N_0$.

Period 1 ($0 \leq t < \mathcal{T}$)

While the fiscal expansion is implemented, the economy follows unstable transitional paths:

$$B(t) = \tilde{B}_1 + D_2 e^{r^{\star}t}, \tag{431}$$

where D_2 is a constant to be determined.

Period 2 ($t \geq \mathcal{T}$)

Once government spending reverts back to its initial level, the economy follows stable paths

$$B(t) = \tilde{B}_2 \tag{432}$$

During the transition period 1, the economy accumulates (or decumulates) foreign assets. Since this period is unstable, it would lead the nation to violate its intertemporal budget constraint. By contrast, the adjustment process taking place in period 2 is stable and must satisfy the economy's intertemporal budget constraint. At the same time, the zero-root problem requires the equilibrium value of marginal utility of wealth to adjust once-and-for-all when the shock hits the economy. So λ remains constant over the periods 1 and 2. The aim of the formal procedure is to calculate the deviation of λ such that the country satisfies one single and overall intertemporal budget constraint, given the new relevant initial condition, $B_{\mathcal{T}}$, accumulated over the unstable period (before the shock is in effect). Therefore, for the country to remain intertemporally solvent, we require:

$$\tilde{B}_2 = B_{\mathcal{T}}. \tag{433}$$

Setting $t = 0$ in solution (431); evaluate at time $t = \mathcal{T}$ and equate (431) and (432):

$$\tilde{B}_1 + D_2 = B_0, \tag{434a}$$

$$\tilde{B}_1 + D_2 e^{r^\star \mathcal{T}} = \tilde{B}_2. \tag{434b}$$

Then, we approximate the steady-state changes with the differentials:

$$\tilde{B}_1 - \tilde{B}_0 \equiv B\left(\bar{\lambda}, G_1^N\right) - B\left(\lambda_0, G_0^N\right) = B_{\bar{\lambda}} \mathrm{d}\bar{\lambda} + B_{G^N} \mathrm{d}G^N, \tag{435a}$$

$$\tilde{B}_2 - \tilde{B}_1 \equiv B\left(\bar{\lambda}, G_2^N\right) - B\left(\bar{\lambda}, G_1^N\right) = -B_{G^N} \mathrm{d}G^N. \tag{435b}$$

By substituting these expressions into (434a) and (434b), we obtain finally

$$D_2 = -B_{\bar{\lambda}} \mathrm{d}\bar{\lambda} - B_{G^N} \mathrm{d}G^N, \tag{436a}$$

$$D_2 e^{r^\star \mathcal{T}} = -B_{G^N} \mathrm{d}G^N. \tag{436b}$$

The two equations (436a)-(436b) jointly determine D_2 and $\bar{\lambda}$.

Solving yields:

$$D_2 = -B_{G^N} e^{-r^\star \mathcal{T}} \mathrm{d}G^N < 0, \tag{437a}$$

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \lambda_{G^N}\left(1 - e^{-r^\star \mathcal{T}}\right) > 0, \tag{437b}$$

where λ_{G^N} corresponds to the change in $\bar{\lambda}$ when the rise in government spending G^N is permanent; it is given by:

$$\lambda_{G^N} = -\frac{B_{G^N}}{B_{\bar{\lambda}}} > 0. \tag{438}$$

Note that (438) coincides with (423). Inserting (423) into (437b) yields the change in the marginal utility of wealth following a temporary rise in government spending on non-tradables G^N:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \frac{P\bar{\lambda}}{Y\chi} \frac{\left[1 + \frac{\omega_C(1-\alpha_C)\alpha_C}{\omega_N}\left(\phi - \sigma_C\right)\eta_{\mu,N}\eta_{N,L^N}\right]}{\Psi}\left(1 - e^{-r^\star \mathcal{T}}\right) > 0. \tag{439}$$

Eq. (439) corresponds to eq. (21) in the text. Importantly, following a temporary fiscal shock, the marginal utility of wealth increases less than after a permanent rise in G^N.

When analyzing the effects of temporary fiscal shocks, we have to consider two periods, i.e., $(0,\mathcal{T})$ and (T,∞), and thus two steady-states. Since we are interested in responses of key macroeconomic variables in the short-run, we derive first the change in non-traded labor over the first period. To do so, we totally differentiate the solution for non-traded labor (410):

$$\left.\frac{\mathrm{d}\tilde{L}_1^N}{\mathrm{d}G^N}\right|_{temp} = \left.\frac{\mathrm{d}L^N(0)}{\mathrm{d}G^N}\right|_{temp} = L_{\bar{\lambda}}^N \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + L_{G^N}^N > 0, \tag{440}$$

where $\mathrm{d}\tilde{L}_1^N/\mathrm{d}G^N = \mathrm{d}L^N(0)/\mathrm{d}G^N$ because non-traded labor adjusts instantaneously to its steady-state associated with period 1 (i.e., period $(0,\mathcal{T})$; moreover, we have $L_{\bar{\lambda}}^N < 0$ and $L_{G^N}^N > 0$ and we used the fact that $\mathrm{d}\tilde{L}_1^N = \tilde{L}_1^N - \tilde{L}_0^N = L^N\left(\bar{\lambda}, G_1^N\right) - L^N\left(\lambda_0, G_0^N\right)$ with $G_1^N - G_0^N = \mathrm{d}G^N$. Because the marginal utility of wealth increases less after a temporary rise in G^N than after a permanent increase in G^N , the negative impact on L^N produced by the wealth effect (which reduces C^N) is smaller. Keeping in mind that L^N rises after a permanent fiscal shock, we can infer from this that non-traded labor increases more following a temporary fiscal shock.

Substituting $L_{\bar{\lambda}}^N < 0$ and $L_{G^N}^N > 0$ given by (411) into (440), the change in non-traded labor in the short-run following a temporary fiscal shock can be rewritten as follows:

$$\left.\frac{\mathrm{d}L^N(0)}{\mathrm{d}G^N}\right|_{temp} = -\frac{\omega_C \alpha_C \sigma_C L^N}{\bar{\lambda}\chi\omega_N} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + \frac{P}{\chi} > 0, \tag{441}$$

where $\chi > 0$ (given by (412)). **Eq. (441) corresponds to eq. (22) in the text.**

Differentiating (407) and inserting (441) yields:

$$\left.\frac{\mathrm{d}P}{\mathrm{d}G^N}\right|_{temp} = \eta_{\mu,N}\eta_{N,L^N} \frac{P}{L^N} \left.\frac{\mathrm{d}L^N}{\mathrm{d}G^N}\right|_{temp} < 0. \tag{442}$$

where $\eta_{\mu,N} < 0$ and $\eta_{N,L^N} > 0$.

Over the unstable period, the economy decumulates traded bonds. To see it formally, differentiate the general solution for traded bonds (431) with respect to time:

$$\dot{B}(t) = r^{\star} D_2 e^{r^{\star} t} < 0, \tag{443}$$

where the sign is stemming from $D_2 < 0$.

References

Glick, and Rogoff (1995) Global Versus Country-Specific Productivity Shocks and the Current Account. *Journal of Monetary Economics* 35, 159-192.

Jaimovich, Nir and Joseph Floetotto (2008) Firm Dynamics, Markup Variations and the Business Cycle. *Journal of Monetary Economics*, 55, 1238-1252.

Schubert, Stefan F., and Stephen J. Turnovsky (2002) The Dynamics of Temporary Policies in a Small Open Economy. *Review of International Economics* 10(4), 604-622.

Yang, Xiaokai, and Ben J. Heijdra (1993) Monopolistic Competifion and Optimum Product Diversity: Comment. *American Economic Review* 83, pp. 295-301.